

## GMAT Quant Topic 1

### General Arithmetic

#### Part A: Overlapping SETS

- Of the films Empty Set Studios released last year, 60% were comedies and the rest were horror films. 75% of the comedies were profitable, but 75% of the horror moves were unprofitable. If the studio made a total of 40 films, and broke even on none of them, how many of their films were profitable?  
18                      19                      20                      21                      22
- At a certain hospital, 75% of the interns receive fewer than 6 hours of sleep and report feeling tired during their shifts. At the same time, 70% of the interns who receive 6 or more hours of sleep report no feelings of tiredness. If 80% of the interns receive fewer than 6 hours of sleep, what percent of the interns report no feelings of tiredness during their shifts?  
6                      14                      19                      20                      81
- All of the students of Music High School are in the band, the orchestra, or both. 80 percent of the students are in only one group. There are 119 students in the band. If 50 percent of the students are in the band only, how many students are in the orchestra only?  
30                      51                      60                      85                      119
- How many attendees are at a convention if 150 of the attendees are neither female nor students, one-sixth of the attendees are female students, two-thirds of the attendees are female, and one-third of the attendees are students?  
300                      450                      600                      800                      900
- Eighty percent of the lights at Hotel California are on at 8 p.m. a certain evening. However, forty percent of the lights that are supposed to be off are actually on and ten percent of the lights that are supposed to be on are actually off. What percent of the lights that are on are supposed to be off?  
22(2/9)%                      16(2/3)%                      11(1/9)%                      10%                      5%
- Of the 645 speckled trout in a certain fishery that contains only speckled and rainbow trout, the number of males is 45 more than twice the number of females. If the ratio of female speckled trout to male rainbow trout is 4:3 and the ratio of male rainbow trout to all trout is 3:20, how many female rainbow trout are there?  
192                      195                      200                      205                      208
- 30% of major airline companies equip their planes with wireless internet access. 70% of major airlines offer passengers free on-board snacks. What is the greatest possible percentage of major airline companies that offer both wireless internet and free on-board snacks?  
21%                      30%                      40%                      50%                      70%
- In country Z, 10% of the people do not have a university diploma but have the job of their choice, and 25% of the people who do not have the job of their choice have a university diploma. If 40% of the people have the job of their choice, what percent of the people have a university diploma?  
35%                      45%                      55%                      65%                      75%
- Seventy percent of the 800 students in School T are male. At least ten percent of the female students in School T participate in a sport. Fewer than thirty percent of the male students in School T do not participate in a sport. What is the maximum possible number of students in School T who do not participate in a sport?  
216                      383                      384                      416                      417
- 75% of the guestrooms at the Stagecoach Inn have a queen-sized bed, and each of the remaining rooms has a king-sized bed. Of the non-smoking rooms, 60% have a queen-sized bed. If 10% of the rooms at the Stagecoach Inn are non-smoking rooms with king-sized beds, what percentage of the rooms permit smoking?  
25%                      30%                      50%                      55%                      75%
- At the end of the day, February 14th, a florist had 120 roses left in his shop, all of which were red, white or pink in color and either long or short-stemmed. A third of the roses were short-stemmed, 20 of which were white and 15 of which were pink. The percentage of pink roses that were short-stemmed equaled the percentage of red roses that were short-stemmed. If none of the long-stemmed roses were white, what percentage of the long-stemmed roses were red?

20%                      25%                      50%                      75%                      80%

12. Some of the people in Town X are left-handed, some are tall, some are both, and some are neither. In Town Y, three times as many people are left-handed as are left-handed in Town X, three times as many people are tall as are tall in Town X, three times as many people are both as are both in Town X, but no one is neither. If the total number of people in Town X is four times greater than the total number of people in Town Y, which of the following could be the number of people in Town X who are neither left-handed nor tall?  
23                      39                      72                      143                      199
13. The waiter at an expensive restaurant has noticed that 60% of the couples order dessert and coffee. However, 20% of the couples who order dessert don't order coffee. What is the probability that the next couple the waiter seats will not order dessert?  
20%                      25%                      40%                      60%                      75%
14. 50% of the apartments in a certain building have windows and hardwood floors. 25% of the apartments without windows have hardwood floors. If 40% of the apartments do not have hardwood floors, what percent of the apartments with windows have hardwood floors?  
10%                      16.66%                      40%                      50%                      83.33%
15. A farmer has an apple orchard consisting of Fuji and Gala apple trees. Due to high winds this year 10% of his trees cross pollinated. The number of his trees that are pure Fuji plus the cross-pollinated ones totals 187, while  $\frac{3}{4}$  of all his trees are pure Fuji. How many of his trees are pure Gala?  
22                      33                      55                      77                      88
16. In a group of 68 students, each student is registered for at least one of three classes – History, Math and English. Twenty-five students are registered for History, twenty-five students are registered for Math, and thirty-four students are registered for English. If only three students are registered for all three classes, how many students are registered for exactly two classes?  
13                      10                      9                      8                      7
17. Each of the 59 members in a high school class is required to sign up for a minimum of one and a maximum of three academic clubs. The three clubs to choose from are the poetry club, the history club, and the writing club. A total of 22 students sign up for the poetry club, 27 students for the history club, and 28 students for the writing club. If 6 students sign up for exactly two clubs, how many students sign up for all three clubs?  
2                      5                      6                      8                      9
18. Each of 435 bags contains at least one of the following three items: raisins, almonds, and peanuts. The number of bags that contain only raisins is 10 times the number of bags that contain only peanuts. The number of bags that contain only almonds is 20 times the number of bags that contain only raisins and peanuts. The number of bags that contain only peanuts is one-fifth the number of bags that contain only almonds. 210 bags contain almonds. How many bags contain only one kind of item?  
256                      260                      316                      320                      350
19. What percent of the students at Jefferson High School study French but not Spanish?  
(1) 30% of all students at Jefferson High School study French.  
(2) 40% of all students at Jefferson High School do not study Spanish.
20. If none of the students are ambidextrous, what percentage of the 20 students in Mr. Henderson's class are left-handed?  
(1) Of the 12 girls in the class, 25% are left-handed.  
(2) 5 of the boys in the class are right-handed.
21. Guests at a recent party ate a total of fifteen hamburgers. Each guest who was neither a student nor a vegetarian ate exactly one hamburger. No hamburger was eaten by any guest who was a student, a vegetarian, or both. If half of the guests were vegetarians, how many guests attended the party?  
(1) The vegetarians attended the party at a rate of 2 students to every 3 non-students, half the rate for non-vegetarians.  
(2) 30% of the guests were vegetarian non-students.
22. To receive a driver license, sixteen year-olds at Culliver High School have to pass both a written and a practical driving test. Everyone has to take the tests, and no one failed both tests. If 30% of the 16 year-olds

who passed the written test did not pass the practical, how many sixteen-year-olds at Culliver High School received their driver license?

(1) There are 188 sixteen year-olds at Culliver High School.

(2) 20% of the sixteen year-olds who passed the practical test failed the written test.

23. At a charity fundraiser, 180 of the guests had a house both in the Hamptons and in Palm Beach. If not everyone at the fundraiser had a house in either the Hamptons or Palm Beach, what is the ratio of the number of people who had a house in Palm Beach but not in the Hamptons to the number of people who had a house in the Hamptons but not in Palm Beach?

(1) One-half of the guests had a house in Palm Beach.

(2) Two-thirds of the guests had a house in the Hamptons

24. Recently Mary gave a birthday party for her daughter at which she served both chocolate and strawberry ice cream. There were 8 boys who had chocolate ice cream, and nine girls who had strawberry. Everybody there had some ice cream, but nobody tried both. What is the maximum possible number of girls who had some chocolate ice cream?

Exactly thirty children attended the party.

Fewer than half the children had strawberry ice cream.

25. Many of the students at the International School speak French or German or both. Among the students who speak French, four times as many speak German as don't. In addition,  $\frac{1}{6}$  of the students who don't speak German do speak French. What fraction of the students speak German?

(1) Exactly 60 students speak French and German.

(2) Exactly 75 students speak neither French nor German.

26. Each member of a pack of 55 wolves has either brown or blue eyes and either a white or a grey coat. If there are more than 3 blue-eyed wolves with white coats, are there more blue-eyed wolves than brown-eyed wolves?

(1) Among the blue-eyed wolves, the ratio of grey coats to white coats is 4 to 3.

(2) Among the brown-eyed wolves, the ratio of white coats to grey coats is 2 to 1.

27. What percentage of the current fourth graders at Liberation Elementary School dressed in costume for Halloween for the past two years in a row (both this year *and* last year)?

(1) 60% of the current fourth graders at Liberation Elementary School dressed in costume for Halloween this year.

(2) Of the current fourth graders at Liberation Elementary School who did not dress in costume for Halloween this year, 80% did not dress in costume last year.

28. Of all the houses on Kermit Lane, 20 have front porches, 20 have front yards, and 40 have back yards. How many houses are on Kermit Lane?

(1) No house on Kermit Lane is without a back yard.

(2) Each house on Kermit Lane that has a front porch does not have a front yard.

29. 55 people live in an apartment complex with three fitness clubs (A, B, and C). Of the 55 residents, 40 residents are members of exactly one of the three fitness clubs in the complex. Are any of the 55 residents members of both fitness clubs A and C but not members of fitness club B?

(1) 2 of the 55 residents are members of all three of the fitness clubs in the apartment complex.

(2) 8 of the 55 residents are members of fitness club B and exactly one other fitness club in the apartment complex.

30. At least 100 students at a certain high school study Japanese. If 4 percent of the students who study French also study Japanese, do more students at the school study French than Japanese?

(1) 16 students at the school study both French and Japanese.

(2) 10 percent of the students at the school who study Japanese also study French.

31. Set A, B, C have some elements in common. if 16 elements are in both A and B, 17 elements are in both A and C, and 18 elements are in both B and C, how many elements do all three of the sets A, B, and C have in common?

(1) Of the 16 elements that are in both A and B, 9 elements are also in C

2) A has 25 elements, B has 30 elements, and C has 35 elements.

32. Of the students who eat in a certain cafeteria, each student either likes or dislikes lima beans and each student either likes or dislikes Brussels sprouts. Of these students,  $\frac{2}{3}$  dislike lima beans; and of those who dislike lima beans,  $\frac{3}{5}$  also dislike Brussels sprouts. How many of the students like Brussels sprout but dislike lima beans?
- (1) 120 students eat in the cafeteria.  
 (2) 40 of the students like lima beans.

### Part B: Percentages

- Two years ago, Arthur gave each of his five children 20 percent of his fortune to invest in any way they saw fit. In the first year, three of the children, Alice, Bob, and Carol, each earned a profit of 50 percent on their investments, while two of the children, Dave and Errol, lost 40 percent on their investments. In the second year, Alice and Bob each earned a 10 percent profit, Carol lost 60 percent, Dave earned 25 percent in profit, and Errol lost all the money he had remaining. What percentage of Arthur's fortune currently remains?  
 93%                  97%                  100%                  107%                  120%
- A car dealership has 40 cars on the lot, 30% of which are silver. If the dealership receives a new shipment of 80 cars, 40% of which are not silver, what percent of the total number of cars are silver?  
 35%                  37.5%                  45%                  47.5%                  50%
- Paul's income is 40% less than Rex's income, Quentin's income is 20% less than Paul's income, and Sam's income is 40% less than Paul's income. If Rex gave 60% of his income to Sam and 40% of his income to Quentin, Quentin's new income would be what fraction of Sam's new income?  
 $\frac{11}{12}$                    $\frac{13}{17}$                    $\frac{13}{19}$                    $\frac{12}{19}$                    $\frac{11}{19}$
- A school's annual budget for the purchase of student computers increased by 60% this year over last year. If the price of student computers increased by 20% this year, then the number of computers it can purchase this year is what percent greater than the number of computers it purchased last year?  
 33.33% 40%                  42%                  48%                  60%
- Boomtown urban planners expect the city's population to increase by 10% per year over the next two years. If that projection were to come true, the population two years from now would be exactly double the population of one year ago. Which of the following is closest to the percent population increase in Boomtown over the last year?  
 20%                  40%                  50%                  65%                  75%
- A retailer bought a shirt at wholesale and marked it up 80% to its initial retail price of \$45. By how many more dollars does he need to increase the price to achieve a 100% markup?  
 1                  2                  3                  4                  5
- A certain NYC taxi driver has decided to start charging a rate of  $r$  cents per person per mile. How much, in dollars, would it cost 3 people to travel  $x$  miles if he decides to give them a 50% discount?  
 $3xr / 2$                    $3x / 200r$                    $3r / 200x$                    $3xr / 200$                    $xr / 600$
- Bob just filled his car's gas tank with 20 gallons of gasohol, a mixture consisting of 5% ethanol and 95% gasoline. If his car runs best on a mixture consisting of 10% ethanol and 90% gasoline, how many gallons of ethanol must he add into the gas tank for his car to achieve optimum performance?  
 $\frac{9}{10}$                   1                   $\frac{10}{9}$                    $\frac{20}{19}$                   2
- Which of the following values is closest to  $\frac{1}{3} + 0.4 + 65\%$ ?  
 1.1                  1.2                  1.3                  1.4                  1.5
- A certain tank is filled to one quarter of its capacity with a mixture consisting of water and sodium chloride. The proportion of sodium chloride in the tank is 40% by volume and the capacity of the tank is 24 gallons. If the water evaporates from the tank at the rate of 0.5 gallons per hour, and the amount of sodium chloride stays the same, what will be the concentration of water in the mixture in 2 hours?  
 43%                  50%                  52%                  54%                  56%
- The useful life of a certain piece of equipment is determined by the following formula:  $u = (8d)/h^2$ , where  $u$  is the useful life of the equipment, in years,  $d$  is the density of the underlying material, in  $\text{g/cm}^3$ , and  $h$  is the number of hours of daily usage of the equipment. If the density of the underlying material is doubled and the

daily usage of the equipment is halved, what will be the percentage increase in the useful life of the equipment?

300%                      400%                      600%                      700%                      800%

12. If  $m > 0$ ,  $y > 0$ , and  $x$  is  $m$  percent of  $2y$ , then, in terms of  $y$ ,  $m$  is what percent of  $x$ ?

$y/200$                        $2y$                        $50y$                        $50/y$                        $5000/y$

13.  $x\%$  of  $y$  is increased by  $x\%$ . What is the result in terms of  $x$  and  $y$ ?

$100xy + x$                        $xy + x/100$   
 $100xy + x/100$                        $100xy + xy/100$                        $xy(x + 100)/10000$

14. The manufacturer's suggested retail price (MSRP) of a certain item is \$60. Store A sells the item for 20 percent more than the MSRP. The regular price of the item at Store B is 30 percent more than the MSRP, but the item is currently on sale for 10 percent less than the regular price. If sales tax is 5 percent of the purchase price at both stores, what is the result when the total cost of the item at Store B is subtracted from the total cost of the item at Store A?

\$0                      \$0.63                      \$1.80                      \$1.89                      \$2.10

15. Two years ago, Sam put \$1,000 into a savings account. At the end of the first year, his account had accrued \$100 in interest bringing his total balance to \$1,100. The next year, his account balance increased by 10%. At the end of the two years, by what percent has Sam's account balance increased from his initial deposit of \$1,000?

19%                      20%                      21%                      22%                      25%

16. The price of a certain painting increased by 20% during the first year and decreased by 15% during the second year. The price of the painting at the end of the 2-year period was what percent of the original price?

102%                      105%                      120%                      135%                      140%

17. If an item that originally sold for  $z$  dollars was marked up by  $x$  percent and then discounted by  $y$  percent, which of the following expressions represents the final price of the item?

$[10,000z + 100z(x - y) - xyz]/10000$                        $[10,000z + 100z(y - x) - xyz]/10000$   
 $[100z(x - y) - xyz]/10000$                        $[100z(y - x) - xyz]/10000$   
 $10000 / [x - y]$

18. A clock store sold a certain clock to a collector for 20 percent more than the store had originally paid for the clock. When the collector tried to resell the clock to the store, the store bought it back at 50 percent of what the collector had paid. The shop then sold the clock again at a profit of 80 percent on its buy-back price. If the difference between the clock's original cost to the shop and the clock's buy-back price was \$100, for how much did the shop sell the clock the second time?

\$270                      \$250                      \$240                      \$220                      \$200

19. 90 students represent  $x$  percent of the boys at Jones Elementary School. If the boys at Jones Elementary make up 40% of the total school population of  $x$  students, what is  $x$ ?

125                      150                      225                      250                      500

20. Cindy has her eye on a sundress but thinks it is too expensive. It goes on sale for 15% less than the original price. Before Cindy can buy the dress, however, the store raises the new price by 25%. If the dress cost \$68 after it went on sale for 15% off, what is the difference between the original price and the final price?

\$0.00                      \$1.00                      \$3.40                      \$5.00                      \$6.80

21. Jennifer has 60 dollars more than Brian. If she were to give Brian  $1/5$  of her money, Brian would have 25% less than the amount that Jennifer would then have. How much money does Jennifer have?

40                      100                      120                      140                      180

22. The average computer price today is \$700. If the average computer price three years ago was 80% of the average computer price today, what was the percentage increase in the average computer price over the past three years?

15%                      20%                      25%                      50%                      80%

23. A small pool filled only with water will require an additional 300 gallons of water in order to be filled to 80% of its capacity. If pumping in these additional 300 gallons of water will increase the amount of water in the pool by 30%, what is the total capacity of the pool in gallons?

- |  |      |      |      |      |      |
|--|------|------|------|------|------|
|  | 1000 | 1250 | 1300 | 1600 | 1625 |
|--|------|------|------|------|------|
24.  $0.2\%$  of  $(3/4)^2 \times (160 \div 10^{-2}) =$   
 $1.8 \times 10^{-3}$                        $1.8 \times 10^{-2}$                       1.8                       $1.8 \times 10$                        $1.8 \times 10^2$
25. 0.35 represents what percent of 0.007?  
0.05%                      0.5%                      5%                      500%                      5000%
26. The price of a certain property increased by 10% in the first year, decreased by 20% in the second year, and increased by 25% in the third year. What was the amount of the dollar decrease in the property price during the second year?  
(1) The price of the property at the end of the third year was \$22,000.  
(2) The decrease in the property price over the first two years was \$2,000 less than the increase in the property price during the third year.
27. A certain salesman's yearly income is determined by a base salary plus a commission on the sales he makes during the year. Did the salesman's base salary account for more than half of the salesman's yearly income last year?  
(1) If the amount of the commission had been 30 percent higher, the salesman's income would have been 10 percent higher last year.  
(2) The difference between the amount of the salesman's base salary and the amount of the commission was equal to 50 percent of the salesman's base salary last year.
28. In the month of June, a street vendor sold 10% more hot dogs than he sold in the month of May. How many total hot dogs did the vendor sell in May and June?  
(1) The vendor sold 27 more hot dogs in June than in May.  
(2) In July, the vendor sold 20% more hot dogs than he sold in May.
29. A sales associate earns a commission of 8% on her first \$10,000 in sales revenue in a given week and a commission of 10% on any additional sales revenue that the associate generates that week. How much sales revenue did the associate generate last week?  
(1) The sales associate earned a total of \$1500 in commission last week.  
(2) Last week, the sales associate was eligible for the 10% commission rate on \$7000 worth of sales.
30. A certain football team played  $x$  games last season, of which the team won exactly  $y$  games. If tied games were not possible, how many games did the team win last season?  
(1) If the team had lost two more of its games last season, it would have won 20 percent of its games for the season.  
(2) If the team had won three more of its games last season, it would have lost 30 percent of its games for the season.
31. In 1994, Company X recorded profits that were 10% greater than in 1993, and in 1993 the company's profits were 20% greater than they were in 1992. What were the company's profits in 1992?  
(1) In 1994, the company's profits were \$100,000 greater than in 1993.  
(2) For every \$3.00 in profits earned in 1992, Company X earned \$3.96 in 1994.
32. All of the furniture for sale at Al's Discount Furniture is offered for less than the manufacturer's suggested retail price (MSRP). Once a year, Al's holds a clearance sale. If Jamie purchased a certain desk during the sale, did she get a discount of more than 50% of Al's regular price for the desk?  
(1) Al's regular price for the desk is 60%, rounded to the nearest percent, of the MSRP of \$2000.  
(2) The sale price was \$601 less than Al's regular price for the desk.
33. The total cost of producing item X is equal to the sum of item X's fixed cost and variable cost. If the variable cost of producing X decreased by 5% in January, by what percent did the total cost of producing item X change in January?  
(1) The fixed cost of producing item X increased by 13% in January.  
(2) Before the changes in January, the fixed cost of producing item X was 5 times the variable cost of producing item X.
34. Of all the attendees at a dinner party, 40% were women. If each attendee arrived at the party either alone or with another attendee of the opposite sex, what percentage of the total number of attendees arrived at the party alone?

- (1) 50% of the male attendees arrived with a woman.  
 (2) 25% of the attendees arriving alone were women.
35. What is 35 percent of  $a^b$ ?  
 (1)  $b$  is 200 percent of  $a$ .  
 (2) 50 percent of  $b$  is  $a$ .
36. Three grades of milk are 1 percent, 2 percent, and 3 percent by volume. If  $x$  gallons of 1 percent grade,  $y$  gallons of 2 percent grade,  $z$  gallons of 3 percent grade are mixed to give  $x+y+z$  gallons of a 1.5 percent grade, what is  $x$  in terms of  $y$  and  $z$ ?
37. Whenever Martin has a restaurant bill with an amount between \$10 and \$99, he calculates the dollar amount of the tip as 2 times the tens digit of the amount of his bill. If the amount of Martin's most recent restaurant bill was between \$10 and \$99, was the tip calculated by Martin on this bill greater than 15 percent of the amount of the bill?  
 (1) The amount of the bill was between \$15 and \$30  
 (2) The tip calculated by Martin was \$8
38. Jack and Mark both received hourly wage increases of 6 percent. After the increases, Jack's hourly wage was how many dollars per hour more than Mark's?  
 (1) Before the wage increases, Jack's hourly wage is \$5 per hour more than Mark's  
 (2) Before the wage increases, the ratio of the Jack's hourly wage to Mark's hourly wage is 4 to 3.
39. A manufacture produced  $x$  percent more video cameras in 1994 than in 1993 and  $y$  percent more video cameras in 1995 than in 1994. If the manufacturer produced 1,000 video cameras in 1993, how many video cameras did the manufacturer produce in 1995?  
 (1)  $xy=20$                       (2)  $x+y+xy/100 = 9.2$
40. What fraction of this year's graduation students at a certain college are males?  
 (1) Of this year's graduation students, 35% of male and 20% of female transferred from another college.  
 (2) Of this year's graduation students, 25% transferred from another college.
41. If  $y$  is greater than 110 percent of  $x$ , is  $y$  greater than 75?  
 (1)  $x > 75$                       (2)  $y - x = 10$
42. At least 10 percent of the people in Country X who are 65 year old or older employed?  
 (1) In country X, 11.3 percent of the population is 65 year old or older  
 (2) In country X, of the population 65 year old or older, 20 percent of the men and 10 percent of the women are employed
43. In 1999 company X's gross profit was what percent of its revenue?  
 (1) In 1999 company X's gross profit was 1/3 of its expenses.  
 (2) In 1999 company X's expenses were 3/4 of its revenue.
44. Henry purchased 3 items during a sale. He received a 20 percent discount off the regular price of the most expensive item of a 10 percent discount off the regular price of each of the other 2 items. Was the total discount of these three items greater than 15 percent of the sum of the regular prices of the 3 items?  
 (1) The regular price of the most expensive item was \$50, and the regular price of the next most expensive item was \$20  
 (2) The regular price of the least expensive item was \$15
45. The rate of a certain chemical reaction is directly proportional to the square of the concentration of chemical A present and inversely proportional to the concentration of chemical B present. If the concentration of chemical B is increased by 100 percent, which of the following is closest to the percent change in the concentration of chemical A required to keep the reaction rate unchanged?
46. Of the 800 employees in a certain company, 70% have serviced more than 10 years. A number of  $y$  of those who have serviced more than 10 years will retire and no fresh employees join in. When is  $y$  if the 10 years employees become 60% of the total employees?
47. Before being simplified, the instructions for computing income tax in Country R were to add 2 percent of one's annual income to the average (arithmetic mean) of 100 units of Country R's currency and 1 percent of one's

annual income. Which of the following represents the simplified formula for computing the income tax, in Country R's currency, for a person in that country whose annual income is A?

50+A/200                      50+3A/100                      50+A/40                      100+A/50                      100+3A/100

48. A certain city with population of 132,000 is to be divided into 11 voting districts, and no district is to have population that is more than 10 percent greater than the population of any other district. What is the minimum possible population that the least population district could have?  
10700                      10800                      10900                      11000                      11100
49. At the end of the first quarter, the share price of a certain mutual fund was 20 percent higher than it was at the beginning of the year. At the end of the second quarter, the share price was 50 percent higher than it was at the beginning of the year. What was the percent increase in the share price from the end of the first quarter to the end of the second quarter?  
20%                      25%                      30%                      33%                      40%
50. A furniture dealer purchased a desk for \$150 and then set the selling price equal to the purchase price plus a markup that was 40 percent of the selling price. If the dealer sold the desk at the selling price, what was the amount of the dealer's gross profit from the purchase and the sale of the desk?  
\$40                      \$60                      \$80                      \$90                      \$100
51. Bobby bought two shares of stock, which sold for \$96 each. If he had a profit of 20 percent on the sale of one of the shares but a loss of 20 percent on the sale of the other share, then on the sale of both shares combined Bobby had:  
a profit of \$10                      a profit of \$8                      a loss of 8  
a loss of 10                      neither profit nor loss
52. In May Mr. Lee's earnings were 60 percent of the Lee family's total income. In June Mr. Lee earned 20 percent more than in May. If the rest of the family's income was the same both months, then, in June, Mrs. Lee's earnings were approximately what percent of the Lee Family's total income?
53. Amy's grade was the 90th percentile of the 80 grades for her class. Of the 100 grades from another class, 19 was higher than Amy's and the rest was lower. If no other grade is the same as Amy's grade, then Amy's grade was what percentile of grades of two class combined.  
72th                      80th                      81th                      85<sup>th</sup>                      92th

### Part C: Work / Rate

1. Machine A and Machine B can produce 1 widget in 3 hours working together at their respective constant rates. If Machine A's speed were doubled, the two machines could produce 1 widget in 2 hours working together at their respective rates. How many hours does it currently take Machine A to produce 1 widget on its own?  
 $\frac{1}{2}$                       2                      3                      5                      6
2. Adam and Brianna plan to install a new tile floor in a classroom. Adam works at a constant rate of 50 tiles per hour, and Brianna works at a constant rate of 55 tiles per hour. If the new floor consists of exactly 1400 tiles, how long will it take Adam and Brianna working together to complete the classroom floor?  
26 hrs. 44 mins.                      26 hrs. 40 mins.                      13 hrs. 20 mins.  
13 hrs. 18 mins.                      12 hrs. 45 mins.
3. A copy machine, working at a constant rate, makes 35 copies per minute. A second copy machine, working at a constant rate, makes 55 copies per minute. Working together at their respective rates, how many copies do the two machines make in half an hour?  
90                      2,700                      4,500                      5,400                      324,000
4. Tom, working alone, can paint a room in 6 hours. Peter and John, working independently, can paint the same room in 3 hours and 2 hours, respectively. Tom starts painting the room and works on his own for one hour. He is then joined by Peter and they work together for an hour. Finally, John joins them and the three of them work together to finish the room, each one working at his respective rate. What fraction of the whole job was done by Peter?  
 $\frac{1}{9}$                        $\frac{1}{6}$                        $\frac{1}{3}$                        $\frac{7}{18}$                        $\frac{4}{9}$



5. Machine A can complete a certain job in  $x$  hours. Machine B can complete the same job in  $y$  hours. If A and B work together at their respective rates to complete the job, which of the following represents the fraction of the job that B will not have to complete because of A's help?  
 $(x - y)/(x + y)$        $x/(y - x)$        $(x + y)/xy$        $y/(x - y)$        $y/(x + y)$
6. Lindsay can paint  $1/x$  of a certain room in 20 minutes. What fraction of the same room can Joseph paint in 20 minutes if the two of them can paint the room in an hour, working together at their respective rates?  
 $1/3x$        $3x/(x - 3)$        $(x - 3)/3x$        $x/(x - 3)$        $(x - 3)/x$
7. One smurf and one elf can build a treehouse together in two hours, but the smurf would need the help of two fairies in order to complete the same job in the same amount of time. If one elf and one fairy worked together, it would take them four hours to build the treehouse. Assuming that work rates for smurfs, elves, and fairies remain constant, how many hours would it take one smurf, one elf, and one fairy, working together, to build the treehouse?  
 $5/7$       1       $10/7$        $12/7$        $22/7$
8. At Supersonic Corporation, the time required for a machine to complete a job is determined by the formula:  $t = \sqrt{w} + \sqrt{(w - 1)}$ , where  $w$  = the weight of the machine in pounds and  $t$  = the hours required to complete the job. If machine A weighs 8 pounds, and machine B weighs 7 pounds, how many hours will it take the two machines to finish one job if they work together?
9. A paint crew gets a rush order to paint 80 houses in a new development. They paint the first  $y$  houses at a rate of  $x$  houses per week. Realizing that they'll be late at this rate, they bring in some more painters and paint the rest of the houses at the rate of  $1.25x$  houses per week. The total time it takes them to paint all the houses under this scenario is what fraction of the time it would have taken if they had painted all the houses at their original rate of  $x$  houses per week?  
 (A)  $0.8(80 - y)$       (B)  $0.8 + 0.0025y$       (C)  $80/y - 1.25$   
 (D)  $80/1.25y$       (E)  $80 - 0.25y$
10. The third-place finisher of the Allen County hot dog eating contest, in which each contestant was given an equal amount of time to eat as many hot dogs as possible, required an average of 15 seconds to consume each hot dog. How many hot dogs did the winner eat?  
 (1) The winner consumed 24 more hot dogs than did the third-place finisher.  
 (2) The winner consumed hot dogs at double the rate of the third-place finisher.
11. On Sunday morning, a printing press printed its newspapers at a constant rate from 1:00 AM to 4:00 AM. How many newspapers did the printing press print on Sunday morning?  
 (1) The printing rate on Saturday morning was twice that of Sunday morning.  
 (2) On Saturday morning, the printing press ran at a constant rate from 1:00 AM to 3:00 AM, stopped for a half hour, and then ran at the same constant rate from 3:30 AM to 5:30 AM, printing a total of 4,000 newspapers.
12. Machine A can fill an order of widgets in  $a$  hours. Machine B can fill the same order of widgets in  $b$  hours. Machines A and B begin to fill an order of widgets at noon, working together at their respective rates. If  $a$  and  $b$  are even integers, is Machine A's rate the same as that of Machine B?  
 (1) Machines A and B finish the order at exactly 4:48 p.m.  
 (2)  $(a + b)^2 = 400$
13. Reserve tank 1 is capable of holding  $z$  gallons of water. Water is pumped into tank 1, which starts off empty, at a rate of  $x$  gallons per minute. Tank 1 simultaneously leaks water at a rate of  $y$  gallons per minute (where  $x > y$ ). The water that leaks out of tank 1 drips into tank 2, which also starts out empty. If the total capacity of tank 2 is twice the number of gallons that remains in tank 1 after one minute, does tank 1 fill up before tank 2?  
 (1)  $zy < 2x^2 - 4xy + 2y^2$   
 (2) The total capacity of tank 2 is less than one-half that of tank 1.
14. Bill can dig a well in  $x$  hours. Carlos can dig the same well in  $y$  hours. If  $q$  is the number of hours that it takes Bill and Carlos to dig the well together, working at their respective rates, is  $q$  an integer?  
 (1)  $x - y = 1$       (2)  $y$  is a nonprime even number.
15. Working alone at its own constant rate, a machine seals  $k$  cartons in 8 hours, and working alone at its own constant rate, a second machine seals  $k$  cartons in 4 hours. If the two machines, each working at its own

constant rate and for the same period of time, together sealed a certain number of cartons, what percent of the cartons were sealed by the machine working at the faster rate?

### Part D: SPEED and DISTANCE

- Bob bikes to school every day at a steady rate of  $x$  miles per hour. On a particular day, Bob had a flat tire exactly halfway to school. He immediately started walking to school at a steady pace of  $y$  miles per hour. He arrived at school exactly  $t$  hours after leaving his home. How many miles is it from the school to Bob's home?  
 $(x + y) / t$                        $2(x + t) / xy$                        $2xyt / (x + y)$   
 $2(x + y + t) / xy$                        $x(y + t) + y(x + t)$
- Lexy walks 5 miles from point A to point B in one hour, then bicycles back to point A along the same route at 15 miles per hour. Ben makes the same round trip, but does so at half of Lexy's average speed. How many minutes does Ben spend on his round trip?  
40                      80                      120                      160                      180
- Triathlete Dan runs along a 2-mile stretch of river and then swims back along the same route. If Dan runs at a rate of 10 miles per hour and swims at a rate of 6 miles per hour, what is his average rate for the entire trip in miles per minute?  
 $1/8$                        $2/15$                        $3/15$                        $1/4$                        $3/8$
- Tom and Linda stand at point A. Linda begins to walk in a straight line away from Tom at a constant rate of 2 miles per hour. One hour later, Tom begins to jog in a straight line in the exact opposite direction at a constant rate of 6 miles per hour. If both Tom and Linda travel indefinitely, what is the positive difference, in minutes, between the amount of time it takes Tom to cover half of the distance that Linda has covered and the amount of time it takes Tom to cover twice the distance that Linda has covered?  
60                      72                      84                      90                      108
- It takes the high-speed train  $x$  hours to travel the  $z$  miles from Town A to Town B at a constant rate, while it takes the regular train  $y$  hours to travel the same distance at a constant rate. If the high-speed train leaves Town A for Town B at the same time that the regular train leaves Town B for Town A, how many more miles will the high-speed train have traveled than the regular train when the two trains pass each other?  
 $\frac{z(y-x)}{x+y}$                        $\frac{z(x-y)}{x+y}$                        $\frac{z(x+y)}{y-x}$                        $\frac{xy(x+y)}{y-x}$                        $\frac{xy(x+y)}{x-y}$
- The 'moving walkway' is a 300-foot long conveyor belt that moves continuously at 3 feet per second. When Bill steps on the walkway, a group of people that are also on the walkway stands 120 feet in front of him. He walks toward the group at a combined rate (including both walkway and foot speed) of 6 feet per second, reaches the group of people, and then remains stationary until the walkway ends. What is Bill's average rate of movement for his trip along the moving walkway?  
2 feet per second                      2.5 feet per second                      3 feet per second  
4 feet per second                      5 feet per second
- John and Jacob set out together on bicycle traveling at 15 and 12 miles per hour, respectively. After 40 minutes, John stops to fix a flat tire. If it takes John one hour to fix the flat tire and Jacob continues to ride during this time, how many hours will it take John to catch up to Jacob assuming he resumes his ride at 15 miles per hour? (consider John's deceleration/acceleration before/after the flat to be negligible)  
3                      3.33                       $3\frac{1}{2}$                       4                       $4\frac{1}{2}$
- Stephanie, Regine, and Brian ran a 20 mile race. Stephanie and Regine's combined times exceeded Brian's time by exactly 2 hours. If nobody ran faster than 8 miles per hour, who could have won the race?  
I. Stephanie                      II. Regine                      III. Brian  
I only                      II only                      III only I or II only                      I, II, or III
- A car traveled from Los Angeles to San Francisco in 6 hours at an average rate of  $x$  miles per hour. If the car returned along the same route at an average rate of  $y$  miles per hour, how long did it take for the car to make the entire round trip, in minutes?
- Deb normally drives to work in 45 minutes at an average speed of 40 miles per hour. This week, however, she plans to bike to work along a route that decreases the total distance she usually travels when driving by

20% . If Deb averages between 12 and 16 miles per hour when biking, how many minutes earlier will she need to leave in the morning in order to ensure she arrives at work at the same time as when she drives?

135                      105                      95                      75                      45

11. Alex and Brenda both stand at point X. Alex begins to walk away from Brenda in a straight line at a rate of 4 miles per hour. One hour later, Brenda begins to ride a bicycle in a straight line in the opposite direction at a rate of  $R$  miles per hour. If  $R > 8$ , which of the following represents the amount of time, in terms of  $R$ , that Alex will have been walking when Brenda has covered twice as much distance as Alex?

$R - 4$                        $R / (R + 4)$                        $R / (R - 8)$                        $8 / (R - 8)$   
 $R^2 - 4$

12. On Monday, Lou drives his ford escort with 28-inch tires, averaging  $x$  miles per hour. On Tuesday, Lou switches the tires on his car to 32-inch tires yet drives to work at the same average speed as on Monday. What is the percent change from Monday to Tuesday in the average number of revolutions that Lou's tires make per second?

Decrease by 14.3%                      Decrease by 12.5%                      Increase by 14.3%  
 Increase by 12.5%                      cannot be determined with the given information.

13. Martha takes a road trip from point A to point B. She drives  $x$  percent of the distance at 60 miles per hour and the remainder at 50 miles per hour. If Martha's average speed for the entire trip is represented as a fraction in its reduced form, in terms of  $x$ , which of the following is the numerator?

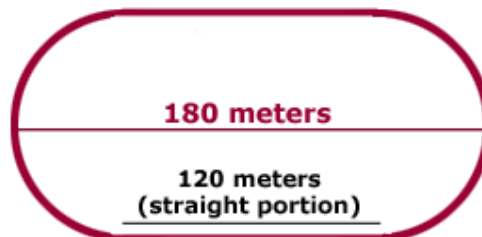
110                      300                      1,100                      3,000                      30,000

14. A not-so-good clockmaker has four clocks on display in the window. Clock #1 loses 15 minutes every hour. Clock #2 gains 15 minutes every hour relative to Clock #1 (i.e., as Clock #1 moves from 12:00 to 1:00, Clock #2 moves from 12:00 to 1:15). Clock #3 loses 20 minutes every hour relative to Clock #2. Finally, Clock #4 gains 20 minutes every hour relative to Clock #3. If the clockmaker resets all four clocks to the correct time at 12 noon, what time will Clock #4 display after 6 actual hours (when it is actually 6:00 pm that same day)?

5:00                      5:34                      5:42                      6:00                      6:24

15. At exactly what time past 7:00 will the minute and hour hands of an accurate working clock be precisely perpendicular to each other for the first time?

16. The figure below represents a track with identical semi-circular ends used for a 4-lap relay race involving two 4-person teams (where each team member runs one complete lap around the track). The table below shows the lap times for each runner on Team A and Team B. Assuming that each runner runs at a constant rate, Team A win the race by how many meters?



Runner	Team A	Team B
1	42 sec	45 sec
2	46 sec	50 sec
3	49 sec	48 sec
4	41 sec	42 sec
Total	178 sec	185 sec

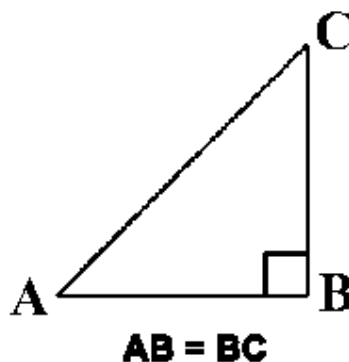
40 meters                       $(40 + 10)$  meters                       $(40 + 20)$  meters  
 $(20 + 10)$  meters                       $(20 + 20)$  meters

17. What is the distance between Harry's home and his office?

(1) Harry's average speed on his commute to work this Monday was 30 miles per hour.  
 (2) If Harry's average speed on his commute to work this Monday had been twice as fast, his trip would have been 15 minutes shorter.

18. Bob and Wendy left home to walk together to a restaurant for dinner. They started out walking at a constant pace of 3 mph. At precisely the halfway point, Bob realized he had forgotten to lock the front door of their home. Wendy continued on to the restaurant at the same constant pace. Meanwhile, Bob, traveling at a new constant speed on the same route, returned home to lock the door and then went to the restaurant to join Wendy. How long did Wendy have to wait for Bob at the restaurant?
- (1) Bob's average speed for the entire journey was 4 mph.
  - (2) On his journey, Bob spent 32 more minutes alone than he did walking with Wendy.
19. If a car traveled from Townsend to Smallville at an average speed of 40 mph and then returned to Townsend later that evening, what was the average speed for the entire trip?
- (1) The return trip took 50% longer than the trip there.
  - (2) The distance from Townsend to Smallville is 165 miles.
20. What was Bill's average speed on his trip of 250 miles from New York City to Boston?
- (1) The trip took Bill 5 hours.
  - (2) At the midpoint of his trip, Bill was going exactly 50 miles per hour.
21. Train A leaves New York for Boston at 3 PM and travels at the constant speed of 100 mph. An hour later, it passes Train B, which is making the trip from Boston to New York at a constant speed. If Train B left Boston at 3:50 PM and if the combined travel time of the two trains is 2 hours, what time did Train B arrive in New York?
- (1) Train B arrived in New York before Train A arrived in Boston.
  - (2) The distance between New York and Boston is greater than 140 miles.
22. Edwin is planning to drive from Boston to New Orleans. By what percent would his travel time be reduced if Edwin decides to split the driving time equally with his friend George, instead of making the trip alone?
- (1) The driving distance from Boston to New Orleans is 1500 miles.
  - (2) George's driving speed is 1.5 times Edwin's driving speed.
23. Trains A and B travel at the same constant rate in opposite directions along the same route between Town G and Town H. If, after traveling for 2 hours, Train A passes Train B, how long does it take Train B to travel the entire distance between Town G and Town H?
- (1) Train B started traveling between Town G and Town H 1 hour after Train A started traveling between Town H and Town G.
  - (2) Train B travels at the rate of 150 miles per hour.

24.



Greg and Brian are both at Point A (above). Starting at the same time, Greg drives to point B while Brian drives to point C. Who arrives at his destination first?

- (1) Greg's average speed is  $\frac{2}{3}$  that of Brian's.
  - (2) Brian's average speed is 20 miles per hour greater than Greg's.
25. If it took Carol  $\frac{1}{2}$  hour to cycle from his house to the library yesterday, was the distance that he cycled greater than 6 miles? (1 mile = 5,280 feet)
- (1) The average speed at which Carlos cycled from his house to the library yesterday was greater than 16 feet per second.
  - (2) The average speed at which Carlos cycled from his house to the library yesterday was less than 18 feet per second

26. How much time did it take a certain car to travel 400 kilometers?  
 (1) The car traveled the first 200 kilometers in 2.5 hours  
 (2) If the car's average speed had been 20 kilometers per hour greater than it was, it would have traveled the 400 kilometers in 1 hours less time than it did.
27. On his trip from Alba to Bento, Julio drove the first  $x$  miles at an average rate of 50 miles per hour and the remaining distant at an average rate of 60 miles per hour, how long did it take Julio to drive the  $x$  miles?  
 (1) On this trip, Julio drove for a total of 10 hours and drove a total of 530 miles  
 (2) On this trip, it took Julio 4 more hour to drive the first  $x$  miles than to drive the remaining distance
28. A hiker walking at a constant rate of 4 miles per hour is passed by a cyclist traveling in the same direction along the same path at a constant rate of 20 miles per hour. The cyclist stops to wait for the hiker 5 minutes after passing her, while the hiker continue to walk at her constant rate. How many minutes must the cyclist wait until the hiker catches up?
29. A boat traveled upstream a distance of 90 miles at an average speed of  $(V-3)$  miles per hour and then traveled the same distance downstream at an average of  $(V+3)$  miles per hour. If the trip upstream took half an hour longer than the trip downstream, how many hours did it take the boat to travel downstream?

### Part E: SI / CI / Population Growth

1. Jolene entered an 18-month investment contract that guarantees to pay 2 percent interest at the end of 6 months, another 3 percent interest at the end of 12 months, and 4 percent interest at the end of the 18 month contract. If each interest payment is reinvested in the contract, and Jolene invested \$10,000 initially, what will be the total amount of interest paid during the 18-month contract?  
 \$506.00      \$726.24      \$900.00      \$920.24      \$926.24
2. Wes works at a science lab that conducts experiments on bacteria. The population of the bacteria multiplies at a constant rate, and his job is to notate the population of a certain group of bacteria each hour. At 1 p.m. on a certain day, he noted that the population was 2,000 and then he left the lab. He returned in time to take a reading at 4 p.m., by which point the population had grown to 250,000. Now he has to fill in the missing data for 2 p.m. and 3 p.m. What was the population at 3 p.m.?  
 50,000      62,500      65,000      86,666      125,000
3. The population of locusts in a certain swarm doubles every two hours. If 4 hours ago there were 1,000 locusts in the swarm, in approximately how many hours will the swarm population exceed 250,000 locusts?  
 6      8      10      12      14
4. An investor purchased a share of non-dividend-paying stock for  $p$  dollars on Monday. For a certain number of days, the value of the share increased by  $r$  percent per day. After this period of constant increase, the value of the share decreased the next day by  $q$  dollars and the investor decided to sell the share at the end of that day for  $v$  dollars, which was the value of the share at that time. How many working days after the investor bought the share was the share sold, if  $r = 100 \left( \sqrt{\frac{v+q}{p}} - 1 \right)$ ?  
 Two working days later.      Three working days later.  
 Four working days later.      Five working days later.  
 Six working days later.
5. A certain investment grows at an annual interest rate of 8%, compounded quarterly. Which of the following equations can be solved to find the number of years,  $x$ , that it would take for the investment to increase by a factor of 16?  
 $16 = (1.02)^{x/4}$        $2 = (1.02)^x$        $16 = (1.08)^{4x}$        $2 = (1.02)^{x/4}$        $1/16 = (1.02)^{4x}$
6. Jim needs \$1,000 to buy a new flat-screen TV. Since he has only \$7, he borrows the remanining balance from his sister Mary. The loan will be repaid in 3 annual installments at an interest rate of 10%, compounded annually. The formula for calculating the monthly payment  $P$  is  $P = (L \times C \times r) / (C - 1)$  where  $L$  = amount of the loan,  $r$  = annual interest rate, and  $C = \text{compounding factor} = (1 + r)^N$  where  $N$  = number of annual payments. How much does Jim have to pay Mary at the end of each of the next 3 years (rounded to the nearest penny)?

\$357.67      \$375.85      \$387.40      \$399.30      \$433.33

7. Louie takes out a three-month loan of \$1000. The lender charges him 10% interest per month compounded monthly. The terms of the loan state that Louie must repay the loan in three equal monthly payments. To the nearest dollar, how much does Louie have to pay each month?  
333                  383                  402                  433                  483
8. Donald plans to invest  $x$  dollars in a savings account that pays interest at an annual rate of 8% compounded quarterly. Approximately what amount is the minimum that Donald will need to invest to earn over \$100 in interest within 6 months?  
\$1500                  \$1750                  \$2000                  \$2500                  \$3000
9. The number of antelope in a certain herd increases every year at a constant rate. If there are 500 antelope in the herd today, how many years will it take for the number of antelope to double?  
(1) Ten years from now, there will be more than ten times the current number of antelope in the herd.  
(2) If the herd were to grow in number at twice its current rate, there would be 980 antelope in the group in two years.
10. A scientist is studying bacteria whose cell population doubles at constant intervals, at which times each cell in the population divides simultaneously. Four hours from now, immediately after the population doubles, the scientist will destroy the entire sample. How many cells will the population contain when the bacteria is destroyed?  
(1) Since the population divided two hours ago, the population has quadrupled, increasing by 3,750 cells.  
(2) The population will double to 40,000 cells with one hour remaining until the scientist destroys the sample.
11. Grace makes an initial deposit of  $x$  dollars into a savings account with a  $z$  percent interest rate, compounded annually. On the same day, Georgia makes an initial deposit of  $y$  dollars into a savings account with a  $z$  percent annual interest rate, compounded quarterly. Assuming that neither Grace nor Georgia makes any other deposits or withdrawals and that  $x$ ,  $y$ , and  $z$  are positive numbers no greater than 50, whose savings account will contain more money at the end of exactly one year?  
(1)  $z = 4$                   (2)  $100y = zx$
12. A certain sum was invested in a high-interest bond for which the interest is compounded monthly. The bond was sold  $x$  number of months later, where  $x$  is an integer. If the value of the original investment doubled during this period, what was the approximate amount of the original investment in dollars?  
(1) The interest rate during the period of investment was greater than 39 percent but less than 45 percent.  
(2) If the period of investment had been one month longer, the final sale value of the bond would have been approximately \$2,744.
15. If a certain culture of bacteria increases by a factor of  $x$  every  $y$  minutes, how long will it take for the culture to increase to ten-thousand times its original amount?  
(1)  $\sqrt[10]{x} = 10$       (2) In two minutes, the culture will increase to one-hundred times its original amount.

## Part F: RATIOS

1. Which of the following fractions is at least twice as great as  $11/50$ ?  
 $2/5$                    $11/34$                    $43/99$                    $8/21$                    $9/20$
2. At the beginning of the year, the ratio of juniors to seniors in high school X was 3 to 4. During the year, 10 juniors and twice as many seniors transferred to another high school, while no new students joined high school X. If, at the end of the year, the ratio of juniors to seniors was 4 to 5, how many seniors were there in high school X at the beginning of the year?  
80                  90                  100                  110                  120
3.  $3/5$  of a certain class left on a field trip.  $1/3$  of the students who stayed behind did **not** want to go on the field trip (all the others did want to go). When another vehicle was located,  $1/2$  of the students who **did** want to go on the field trip but had been left behind were able to join. What fraction of the class ended up going on the field trip?  
 $1/2$                    $2/3$                    $11/15$                    $23/30$                    $4/5$
4. The ratio of boys to girls in Class A is 3 to 4. The ratio of boys to girls in Class B is 4 to 5. If the two classes were combined, the ratio of boys to girls in the combined class would be 17 to 22. If the number of boys in

Class B is one less than the number of boys in Class A, and if the number of girls in Class B is two less than the number of girls in Class A, how many girls are in Class A?

8                      9                      10                      11                      12

5. John's front lawn is  $\frac{1}{3}$  the size of his back lawn. If John mows  $\frac{1}{2}$  of his front lawn and  $\frac{2}{3}$  of his back lawn, what fraction of his lawn is left unmowed?  
 $\frac{1}{6}$                        $\frac{1}{3}$                        $\frac{3}{8}$                        $\frac{1}{2}$                        $\frac{5}{8}$
6. At Jefferson Elementary School, the number of teachers and students (kindergarten through sixth grade) totals 510. The ratio of students to teachers is 16 to 1. Kindergarten students make up  $\frac{1}{5}$  of the student population and fifth and sixth graders account for  $\frac{1}{3}$  of the remainder. Students in first and second grades account for  $\frac{1}{4}$  of all the students. If there are an equal number of students in the third and fourth grades, then the number of students in third grade is how many greater or fewer than the number of students in kindergarten?  
12 greater                      17 fewer                      28 fewer                      36 fewer                      44 fewer
7. A certain galaxy is known to comprise approximately  $4 \times 10^{11}$  stars. Of every 50 million of these stars, one is larger in mass than our sun. Approximately how many stars in this galaxy are larger than the sun?  
800                      1,250                      8,000                      12,000                      80,000
8. A lemonade stand sold only small and large cups of lemonade on Tuesday.  $\frac{3}{5}$  of the cups sold were small and the rest were large. If the large cups were sold for  $\frac{7}{6}$  as much as the small cups, what fraction of Tuesday's total revenue was from the sale of large cups?  
 $\frac{7}{16}$                        $\frac{7}{15}$                        $\frac{10}{21}$                        $\frac{17}{35}$                        $\frac{1}{2}$
9. Miguel is mixing up a salad dressing. Regardless of the number of servings, the recipe requires that  $\frac{5}{8}$  of the finished dressing mix be olive oil,  $\frac{1}{4}$  vinegar, and the remainder an even mixture of salt, pepper and sugar. If Miguel accidentally doubles the vinegar and forgets the sugar altogether, what proportion of the botched dressing will be olive oil?  
 $\frac{15}{29}$                        $\frac{5}{8}$                        $\frac{5}{16}$                        $\frac{1}{2}$                        $\frac{13}{27}$
10. Harold and Millicent are getting married and need to combine their already-full libraries. If Harold, who has  $\frac{1}{2}$  as many books as Millicent, brings  $\frac{1}{3}$  of his books to their new home, then Millicent will have enough room to bring  $\frac{1}{2}$  of her books to their new home. What fraction of Millicent's old library capacity is the new home's library capacity?  
 $\frac{1}{2}$                        $\frac{2}{3}$                        $\frac{3}{4}$                        $\frac{4}{5}$                        $\frac{5}{6}$
11. In a certain pet shop, the ratio of dogs to cats to bunnies in stock is 3 : 5 : 7. If the shop carries 48 cats and bunnies total in stock, how many dogs are there?  
12                      13                      14                      15                      16
12. A foreign language club at Washington Middle School consists of  $n$  students,  $\frac{2}{5}$  of whom are boys. All of the students in the club study exactly one foreign language.  $\frac{1}{3}$  of the girls in the club study Spanish and  $\frac{5}{6}$  of the remaining girls study French. If the rest of the girls in the club study German, how many girls in the club, in terms of  $n$ , study German?  
 $\frac{2n}{5}$                        $\frac{n}{3}$                        $\frac{n}{5}$                        $\frac{2n}{15}$                        $\frac{n}{15}$
13. A certain ball team has an equal number of right- and left-handed players. On a certain day, two-thirds of the players were absent from practice. Of the players at practice that day, one-third were left handed. What is the ratio of the number of right-handed players who were not at practice that day to the number of left-handed players who were not at practice?  
 $\frac{1}{3}$                        $\frac{2}{3}$                        $\frac{5}{7}$                        $\frac{7}{5}$                        $\frac{3}{2}$
14. Bag A contains red, white and blue marbles such that the red to white marble ratio is 1:3 and the white to blue marble ratio is 2:3. Bag B contains red and white marbles in the ratio of 1:4. Together, the two bags contain 30 white marbles. How many red marbles could be in bag A?  
1                      3                      4                      6                      8
15. The ratio by weight, measured in pounds, of books to clothes to electronics in Jorge's suitcase initially stands at 8 to 5 to 3. Jorge then removes 4 pounds of clothing from his suitcase, thereby doubling the ratio of books to clothes. Approximately how much do the electronics in the suitcase weigh, to the nearest pound?  
3                      4                      5                      6                      7

16. Joe, Bob and Dan worked in the ratio of 1:2:4 hours, respectively. How many hours did Bob work?  
(1) Together, Joe, Bob and Dan worked a total of 49 hours.  
(2) Dan worked 21 hours more than Joe.
17. In 2003 Acme Computer priced its computers five times higher than its printers. What is the ratio of its gross revenue for computers and printers respectively in the year 2003?  
(1) In the first half of 2003 it sold computers and printers in the ratio of 3:2, respectively, and in the second half in the ratio of 2:1.  
(2) It sold each computer for \$1000.
18. If Pool Y currently contains more water than Pool X, and if Pool X is currently filled to  $\frac{2}{7}$  of its capacity, what percent of the water currently in Pool Y needs to be transferred to Pool X if Pool X and Pool Y are to have equal volumes of water?  
(1) If all the water currently in Pool Y were transferred to Pool X, Pool X would be filled to  $\frac{6}{7}$  of its capacity.  
(2) Pool X has a capacity of 14,000 gallons.
19. Three business partners shared all the proceeds from the sale of their privately held company. If the partner with the largest share received exactly  $\frac{5}{8}$  of the total proceeds, how much money did the partner with the smallest share receive from the sale?  
(1) The partner with the smallest share received from the sale exactly  $\frac{1}{5}$  the amount received by the partner with the second largest share.  
(2) The partner with the second largest share received from the sale exactly half of the two million dollars received by the partner with the largest share.
20. In a piggy bank filled with only pennies, nickels, and dimes, what is the ratio of pennies to dimes?  
(1) The ratio of nickels to dimes is three to two.  
(2) There is exactly \$7 in the piggy bank.
21. In a certain solution consisting of only two chemicals, for every 3 milliliters of Chemical A, there are 7 milliliters of Chemical B. After 10 milliliters of Chemical C are added to this solution, what is the ratio of the quantities of Chemical A to Chemical C?  
(1) Before Chemical C was added, there were 50 milliliters of solution.  
(2) After Chemical C was added, there were 60 milliliters of solution.
22. On a certain sight-seeing tour, the ratio of the number of women to the number of children was 5 to 2. What was the number of men on the sight-seeing tour?  
(1) On the sight-seeing tour, the ratio of the number of children to the number of men was 5 to 11.  
(2) The number of women on the sight-seeing tour was less than 30.
23. Each employee of Company Z is an employee of either Division X or Division Y, but not both. If each division has some part-time employees, is the ratio of the number of full-time employees to the number of part-time employees greater for Division X than for Company Z?  
(1) The ratio of the number of full-time employees to the number of part-time employees is less for Division Y than for Company Z.  
(2) More than half the full-time employees of Company Z are employees of Division X, and more than half of the part-time employees of Company Z are employees of Division Y.
24. Of the 60 animals on a certain farm,  $\frac{2}{3}$  are either pigs or cows. How many of the animals are cows?  
(1) the farm has more than twice as many cows as it has pigs.  
(2) the farm has more than 12 pigs
25. Malik's recipe for 4 servings of a certain dish requires  $\frac{3}{2}$  cups of pasta. According to this recipe, what is the number of cups of pasta that Malik will use the next time he prepares this dish?  
(1) The next time he prepares this dish, Malik will make half as many servings as he did the last time he prepared the dish.  
(2) Malik used 6 cups of pasta the last time he prepared this dish.



## GMAT Quant Topic 2

### Statistics

#### Mean

- The table below provides revenues of a certain company in 2002 and 2003. By what percent did the average quarterly revenue change from 2002 to 2003?

Quarter	Quarterly revenues, MM USD	
	2002	2003
1 <sup>st</sup>	13	17
2 <sup>nd</sup>	15	18
3 <sup>rd</sup>	16	17
4 <sup>th</sup>	16	20

- During 2005, a company produced an average of 2,000 products per month. How many products will the company need to produce from 2006 through 2008 in order to increase its monthly average for the period from 2005 through 2008 by 200% over its 2005 average?  
(A) 148,000      (B) 172,000      (C) 200,000      (D) 264,000      (E) 288,000

- After his first semester in college, Thomas is applying for a scholarship that has a minimum Grade Point Average (GPA) requirement of 3.5. The point values of pertinent college grades are given in the table below. If Thomas took 5 courses, each with an equal weight for GPA calculations, and received two grades of A-, one grade of B+, and one grade of B, what is the lowest grade that Thomas could receive for his fifth class to qualify for the scholarship?

#### Point Values of Select Grades

Grade	A	A-	B+	B	B-	C+	C	C-
Value	4	3.7	3.3	3	2.7	2.3	2	1.7

(A) A      (B) B+      (C) B      (D) B-      (E) C+

- A certain portfolio consisted of 5 stocks, priced at \$20, \$35, \$40, \$45, and \$70, respectively. On a given day, the price of one stock increased by 15%, while the price of another stock decreased by 35% and the prices of the remaining three remained constant. If the average price of a stock in the portfolio rose by approximately 2%, which of the following could be the prices of the shares that remained constant?  
(A) \$20, \$35, and \$70      (B) \$20, \$45, and \$70      (C) \$20, \$35, and \$40  
(D) \$35, \$40, and \$70      (E) \$35, \$40, and \$45
- If John makes a contribution to a charity fund at school, the average contribution size will increase by 50%, reaching \$75 per person. If there were 5 other contributions made before John's, what is the size of his donation?  
(A) \$100      (B) \$150      (C) \$200      (D) \$250      (E) \$450
- What is the minimum percentage increase in the mean of set X {-4, -1, 0, 6, 9} if its two smallest elements are replaced with two different primes?  
(A) 25%      (B) 50%      (C) 75%      (D) 100%      (E) 200%
- If every member of set X {-14, -12, 17, 28, 41, Z} is multiplied by number N, by what percent will the mean M of the set increase?  
(1) Z = 60      (2) N = Z / M
- Which of the following series of numbers, if added to the set {1, 6, 11, 16, 21}, will not change the set's mean?  
I. 1.5, 7.11 and 16.89      II. 5.36, 10.7 and 13.24      III. -21.52, 23.3, 31.22  
(A) I only      (B) II only      (C) III only      (D) I and III only      (E) None
- If numbers N and K are added to set X {2, 8, 10, 12}, its mean will increase by 25%. What is the value of  $N^2 + 2NK + K^2$ ?  
(A) 28      (B) 32      (C) 64      (D) 784      (E) 3600
- Set X consists of different positive numbers arranged in ascending order: K, L, M, 5, 7. If K, L and M are consecutive integers, what is the arithmetic mean of set X?

- (1) The product  $K \times L \times M$  is a multiple of 6  
 (2) There are at least 2 prime numbers among  $K$ ,  $L$  and  $M$
11. A group of men and women gathered to compete in a marathon. Before the competition, each competitor was weighed and the average weight of the female competitors was found to be 120 lbs. What percentage of the competitors were women?  
 (1) The average weight of the men was 150 lb.  
 (2) The average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women.
12. The mean of  $(54,820)^2$  and  $(54,822)^2 =$   
 (A)  $(54,821)^2$  (B)  $(54,821.5)^2$  (C)  $(54,820.5)^2$  (D)  $(54,821)^2 + 1$  (E)  $(54,821)^2 - 1$
13. Set  $S$  consists of integers 7, 8, 10, 12, and 13. If integer  $n$  is included in the set, the average (arithmetic mean) of set  $S$  will increase by 20%. What is the value of integer  $n$ ?  
 10                      12                      16                      22                      24
14. A convenience store currently stocks 48 bottles of mineral water. The bottles have two sizes of either 20 or 40 ounces each. The average volume per bottle the store currently has in stock is 35 ounces. How many 40 ounce bottles must be sold for the average volume per bottle to be reduced to 25 ounces if no 20 ounce bottles are sold?  
 10                      20                      30                      32                      34
15. Last year, the five employees of Company X took an average of 16 vacation days each. What was the average number of vacation days taken by the same employees this year?  
 (1) Three employees had a 50% increase in their number of vacation days, and two employees had a 50% decrease.  
 (2) Three employees had 10 more vacation days each, and two employees had 5 fewer vacation days each.
16. In a room of men and women, the average weight of the women is 120 lbs, and the average weight of the men is 150 lbs. What is the average weight of a person in the room?  
 (1) There are twice as many men as women.  
 (2) There are a total of 120 people in the room.
17. If set  $R$  contains the consecutive integers from -5 to -1, what is the mean of set  $R$ ?  
 -5                      -3                      0                      3                      5
18. In Greenville last July, what was the average (arithmetic mean) home sale price?  
 (1) In Greenville last July, there were 100 homes sold for a total of \$51 million.  
 (2) In Greenville last July, condominiums accounted for 60% of the home sales, and the average condominium price was \$450,000.
19.  $x$ ,  $y$ , and  $z$  are positive integers. The average (arithmetic mean) of  $x$ ,  $y$ , and  $z$  is 11. If  $z$  is two greater than  $x$ , which of the following must be true?  
 I.  $x$  is even                      II.  $y$  is odd                      III.  $z$  is odd  
 I only                      II only                      III only                      I and II only                      I and III only
20. Set  $A$  contains the consecutive integers ranging from  $x$  to  $y$ , inclusive. If the number of integers in set  $A$  that are less than 75 is equal to the number of integers that are greater than 75, what is the value of  $3x + 3y$ ?  
 225                      300                      372                      450                      528
21. In a work force, the employees are either managers or directors. What is the percentage of directors?  
 (1) the average salary for manager is \$5,000 less than the total average salary.  
 (2) the average salary for directors is \$15,000 more than the total average salary.
22. In the first week of last month, Company X realized an average wholesale profit of \$5304 per day from the sale of  $q$  units of Product Y. Which of the following CANNOT be the difference between Product Y's sale price and cost per unit?  
 \$3                      \$4                      \$7                      \$11                      \$51

23. A certain bank has ten branches. What is the total amount of assets under management at the bank?
- (1) There is an average of 400 customers per branch. When each branch's average assets under management per customer is computed, these values are added together and this sum is divided by 10. The result is \$400,000 per customer.
  - (2) The bank has a total of 4,000 customers. When the total assets per branch are added up, each branch is found to manage, on average, 160 million dollars in assets.
24. Three baseball teams, A, B, and C, play in a seasonal league. The ratio of the number of players on the three teams is 2:5:3, respectively. Is the average number of runs scored per player across all three teams collectively greater than 22?
- (1) The average number of runs scored per player for each of the three teams, A, B, and C, is 30, 17, and 25, respectively.
  - (2) The total number of runs scored across all three teams collectively is at least 220.
25. The average score of  $x$  number of exams is  $y$ . When an additional exam of score  $z$  is added in, does the average score of the exams increase by 50%?
- (1)  $3x = y$
  - (2)  $2z - 3y = xy$
26. A new cell phone plan is offering pricing based on average monthly use. Brandon and Jodie are comparing their average use to determine the best plan for them. Brandon's average monthly usage in 2001 was  $q$  minutes. Was this less than, greater than, or equal to Jodie's 2001 average monthly usage, in minutes?
- (1) From January to August 2001, Jodie's average monthly usage was  $1.5q$  minutes.
  - (2) From April to December 2001, Jodie's average monthly usage was  $1.5q$  minutes.
27. On Jane's credit card account, the average daily balance for a 30-day billing cycle is average (arithmetic mean) of the daily balances at the end of the 30 days. At the beginning of a certain 30-day billing cycle, Jane's credit card account had a balance of \$600. Jane made a payment of \$300 on the account during the billing cycle. If no other amounts were added to or subtracted from the account during the billing cycle, what was average daily balance on Jane's account for the billing cycle?
- (1) Jane's payment was credited on the 21st day of the billing cycle.
  - (2) The average daily balance through the 25th day of the billing cycle was \$540.
28. L spends total \$6.00 for one kind of D and one kind of C. How many D did he buy?
- (1) the price of 2D was \$0.10 less than the price of 3C
  - (2) the average price of 1 D and 1 C was \$0.35
29.  $x$ ,  $y$ , and  $z$  are consecutive integers, and  $x < y < z$ . What is the average of  $x$ ,  $y$ , and  $z$ ?
- (1)  $x = 11$
  - (2) The average of  $y$  and  $z$  is 12.5.

## Median

- Set A consists of numbers  $\{-2, 27.5, -6, 18.3, 9\}$  and set B consists of numbers  $\{-199, 0.355, 19.98, 10, 201, 16\}$ . The median of set B is how much greater than the median of set A?
- Which of the following could be the median of a set consisting of 6 different primes?  
(A) 2                      (B) 3                      (C) 9.5                      (D) 12.5                      (E) 39
- The median annual household income in a certain community of 21 households is \$50,000. If the mean income of a household increases by 10% per year over the next 2 years, what will the median income in the community be in 2 years?  
(A) \$50,000      (B) \$60,000      (C) \$60,500      (D) \$65,000      (E) Cannot get
- What is the median of set A  $\{-8, 15, -9, 4, N\}$ ?  
(1)  $N$  is a prime and  $N^6$  is even                      (2)  $2N + 14 < 20$
- T is a set of  $y$  integers, where  $0 < y < 7$ . If the average of Set T is the positive integer  $x$ , which of the following could NOT be the median of Set T?  
(A) 0                      (B)  $x$                       (C)  $-x$                       (D)  $y/3$                       (E)  $2y/7$

6.  $a$ ,  $b$ , and  $c$  are integers and  $a < b < c$ .  $S$  is the set of all integers from  $a$  to  $b$ , inclusive.  $Q$  is the set of all integers from  $b$  to  $c$ , inclusive. The median of set  $S$  is  $(3/4)b$ . The median of set  $Q$  is  $(7/8)c$ . If  $R$  is the set of all integers from  $a$  to  $c$ , inclusive, what fraction of  $c$  is the median of set  $R$ ?  
 (A)  $3/8$  (B)  $1/2$  (C)  $11/16$  (D)  $5/7$  (E)  $3/4$
7. Jim Broke's only source of income comes from his job as a question writer. In this capacity, Jim earns a flat salary of \$200 per week plus a fee of \$9 for every question that he writes. Every year, Jim takes exactly two weeks of unpaid vacation to visit his uncle, a monk in Tibet, and get inspired for the next year. If a regular year consists of 52 weeks and the number of questions that Jim wrote in each of the past 5 years was an odd number greater than 20, which of the following could be Jim's median annual income over the past 5 years?  
 (A) \$22,474 (B) \$25,673 (C) \$27,318 (D) \$28,423 (E) \$31,227
8. Set A, Set B, and Set C each contain only positive integers. If Set A is composed entirely of all the members of Set B plus all the members of Set C, is the median of Set B greater than the median of Set A?  
 (1) The mean of Set A is greater than the median of Set B.  
 (2) The median of Set A is greater than the median of Set C.
9. If  $x$  and  $y$  are unknown positive integers, is the mean of the set  $\{6, 7, 1, 5, x, y\}$  greater than the median of the set?  
 (1)  $x + y = 7$  (2)  $x - y = 3$
10. Given the ascending set  $\{x, x, y, y, y, y\}$ . What is greater, the median or the mean?
11. There is a set of numbers in ascending order:  $\{y - x, y, y, y, y, x, x, x, x + y\}$ . If the mean is 9, and the median is 7, what is  $x$ ?
12. During a behavioral experiment in a psychology class, each student is asked to compute his or her lucky number by raising 7 to the power of the student's favorite day of the week (numbered 1 through 7 for Monday through Sunday respectively), multiplying the result by 3, and adding this to the doubled age of the student in years, rounded to the nearest year. If a class consists of 28 students, what is the probability that the median lucky number in the class will be a non-integer?  
 (A) 0% (B) 10% (C) 20% (D) 30% (E) 40%
13. Given the ascending set of positive integers  $\{a, b, c, d, e, f\}$ , is the median greater than the mean?  
 (1)  $a + e = (3/4)(c + d)$  (2)  $b + f = (4/3)(c + d)$
14. For the set of terms  $[x, y, x + y, x - 4y, xy, 2y]$ , if  $y > 6$  and the mean of the set equals  $y + 3$ , then the median must be  
 $(x + y) / 2$        $y + 3$        $y$        $3y/2$        $(x/2) + y$
15. What is the median value of the set  $R$ , if for every term in the set,  $R_n = R_{n-1} + 3$ ?  
 (1) The first term of set  $R$  is 15. (2) The mean of set  $R$  is 36.
16. Peter, Paul, and Mary each received a passing score on his/her history midterm. The average (arithmetic mean) of the three scores was 78. What was the median of the three scores?  
 (1) Peter scored a 73 on his exam. (2) Mary scored a 78 on her exam.
17. Set A: 3,  $x$ , 8, 10      Set B: 4,  $y$ , 9, 11.      The terms of each set above are given in ascending order. If the median of Set A is equal to the median of Set B, what is the value of  $y - x$ ?  
 -2      -1      0      1      2

18. Set  $S$  includes elements  $\{8, 2, 11, x, 3, y\}$  and has a mean of 7 and a median of 5.5. If  $x < y$ , then which of the following is the maximum possible value of  $x$ ?
- 0                      1                      2                      3                      4
19. If set  $S$  consist of the numbers 1, 5, -2, 8, and  $n$ , is  $0 < n < 7$ ?
- (1) the median of the numbers in  $S$  is less than 5.  
 (2) the median of the numbers in  $S$  is greater than 1.
20. Set  $S$  consists of five consecutive integers, and set  $T$  consists of seven consecutive integers. Is the median of the numbers in set  $S$  equal to the median of the numbers in set  $T$ ?
- (1) The median of the numbers in set  $S$  is 0.  
 (2) The sum of the numbers in set  $S$  is equal to the sum of the numbers in set  $T$ .
21. The temperatures in Celsius recorder at 6 in the morning in various parts of a certain country were 10, 5, -2, -1, -5 and 15. What is the median of these temperatures?
- 2                      -1                      2                      3                      5
22. A student worked for 20 days. For each of the amounts shown in the first row of the table, the second row gives the number of days that the students earned that amount. What is the median amount of money that the student earned per day for the 20 days?

Amount earned per day	\$96	\$84	\$80	\$70	\$48
Number of days	4	7	4	3	2

23. Score                      Number and Interval of Scores
- |       |    |
|-------|----|
| 50-59 | 2  |
| 60-69 | 10 |
| 70-79 | 16 |
| 80-89 | 27 |
| 90-99 | 18 |
- The table above shows the distribution of test scores for a group of management trainees, which score interval contains the median of the 73 scores?
- A. 60-69                      B. 70-79                      C. 80-89                      D. 90-99                      E. Can't get
24. Last month 15 homes were sold in Town X. The average (arithmetic mean) sale price of the homes was \$ 150,000 and the median sale price was \$130,000. Which of the following statement must be true?
- I. at least one of the homes was sold for more than \$165,000  
 II. at least one of the homes was sold for more than \$130,000 and less than \$150,000  
 III. at least one of the homes was sold for less than \$130,000
25. Five pieces of wood have an average (arithmetic mean) length of 124 centimeters and a median length of 140 centimeters. What is the maximum possible length in centimeters of the shortest piece of wood?
- 90                      100                      110                      130                      140
26. Amy's grade was the 90th percentile of the 80 grades for her class. Of the 100 grades from another class, 19 was higher than Amy's and the rest was lower. If no other grade is the same as Amy' grade, then Army's grade was what percentile of grades of two class combined.
- 72<sup>nd</sup>                      80<sup>th</sup>                      81<sup>st</sup>                      85<sup>th</sup>                      92<sup>nd</sup>

27.

Ann	\$450,000
Bob	\$360,000
Cal	\$190,000
Dot	\$210,000
Ed	\$680,000

The table above shows the total sales recorded in July for the five salespeople. It was discovered that one of Cal's sales was incorrectly recorded as one of Ann's sales. After this error was corrected, Ann's total sales were still higher than Cal's total sales, and the median of 5 sales totals was \$330,000. What was the value of the incorrectly recorded sale?

## Mode

- Set A, B, and C consist of the following elements:  
A {0, 3, 4, 2, 0, 4, 7, 8, 4, 17}      B {20, 12, -7, -9, -5, -7, 11, -5, 68}      C {-1.5, 0, 1.5}. If Z is defined as the sum of modes of sets A, B, and C, what is the value of Z?
- The mode of a set of integers is x. What is the difference between the median of this set of integers and x?  
(1) The difference between any two integers in the set is less than 3.  
(2) The average of the set of integers is x.

## Range

- If set X contains numbers {-21, 6, 19, 126, 1000} and set Y contains numbers {-21, 990, 993, 996.19, 997.05, 999, 1000}, what is the difference between the ranges of set X and set Y?
- Set X consists of prime numbers {3, 11, 7, K, 17, 19}. If integer Y represents the product of all elements in set X and if 11Y is an even number, what is the range of set X?  
(A) 14   (B) 16      (C) 17      (D) 20      (E) 26
- What could be the range of a set consisting of odd multiples of 7?  
(A) 21      (B) 24      (C) 35      (D) 62      (E) 70
- What is the range of a set consisting of the first 100 multiples of 7 that are greater than 70?  
(A) 693      (B) 700      (C) 707      (D) 777      (E) 847
- Set X consists of all two-digit primes and set Y consists of all positive odd multiples of 5 less than 100. If the two sets are combined into one, what will be the range of the new set?  
(A) 84      (B) 89      (C) 90      (D) 92      (E) 95
- At a business school conference with 100 attendees, are there any students of the same age (rounded to the nearest year) who attend the same school?  
(1) The range of ages of the participants is 22 to 30, inclusive  
(2) Participants represent 10 business schools
- Set A consists of integers {3, -8, Y, 19, -6} and Set B consists of integers {K, -3, 0, 16, -5, 9}. Number L represents the median of Set A, number M represents the mode of set B, and number  $Z = L^M$ . If Y is an integer greater than 21, for what value of K will Z be a divisor of 26?  
(A) -2   (B) -1      (C) 0      (D) 1      (E) 2
- If two elements are dropped from set X {-10, -8, 0, 6, 7}, what will be the percentage change in its mean?  
(1) The median of the set will remain the same   (2) The range of the set will decrease by 3
- If a randomly selected non-negative single digit integer is added to set X {2, 3, 7, 8}, what is the probability that the median of the set will increase while its range will remain the same?  
(A) 20%      (B) 30%      (C) 40%      (D) 50%      (E) 60%
- Set A consists of all positive integers less than 100; Set B consists of 10 integers, the first four of which are 2, 3, 5, and 7. What is the difference between the median of Set A and the range of Set B?  
(1) All numbers in Set B are prime numbers;  
(2) Each element in Set B is divisible by exactly two factors.
- Set A consists of 8 distinct prime numbers. If x is equal to the range of set A and y is equal to the median of set A, is the product xy even?  
(1) The smallest integer in the set is 5.   (2) The largest integer in the set is 101.
- If set S = {7, y, 12, 8, x, 9}, is x + y less than 18?  
(1) The range of set S is less than 9.  
(2) The average of x and y is less than the average of set S.

13. The GMAT is scored on a scale of 200 to 800 in 10 point increments. (Thus 410 and 760 are real GMAT scores but 412 and 765 are not). A first-year class at a certain business school consists of 478 students. Did any students of the same gender in the first-year class who were born in the same-named month have the same GMAT score?
- (1) The range of GMAT scores in the first-year class is 600 to 780.  
 (2) 60% of the students in the first-year class are male.
14.  $S$  is a set of positive integers. The average of the terms in  $S$  is equal to the range of the terms in  $S$ . What is the sum of all the integers in  $S$ ?
- (1) The range of  $S$  is a prime number that is less than 11 and is not a factor of 10.  
 (2)  $S$  is composed of 5 different integers.
15. If  $S$  is a finite set of consecutive even numbers, is the median of  $S$  an odd number?
- (1) The mean of set  $S$  is an even number.  
 (2) The range of set  $S$  is divisible by 6.
16. 10 students took a chemistry exam that was graded on a scale of 0 to 100. Five of the students were in Dr. Adams' class and the other five students were in Dr. Brown's class. Is the median score for Dr. Adams' students greater than the median score for Dr. Brown's students?
- (1) The range of scores for students in Dr. Adams' class was 40 to 80, while the range of scores for students in Dr. Brown's class was 50 to 90.  
 (2) If the students are paired in study teams such that each student from Dr. Adams' class has a partner from Dr. Brown's class, there is a way to pair the 10 students such that the higher scorer in each pair is one of Dr. Brown's students.
17.  $x$  is an integer greater than 7. What is the median of the set of integers from 1 to  $x$  inclusive?
- (1) The average of the set of integers from 1 to  $x$  inclusive is 11.  
 (2) The range of the set of integers from 1 to  $x$  inclusive is 20.
18. 

Stock	number of shares
v	68
w	112
x	56
y	94
z	45
- The table shows the number of shares of each of the 5 stocks owned by Mr. Sami. If Mr Sami was to sell 20 shares of Stock X and buy 24 shares of stock y, what would be the increase in range of the number of shares of the 5 stocks owned by Mr Sami?
- 4                      6                      9                      15                      20
19. The numbers of books read by 7 students last year were 10, 5,  $p$ ,  $q$ ,  $r$ , 29 and 20. What was the range of the numbers of books read by the 7 students last year?
- (1)  $5 < p < q$                       (2)  $p < r < 15$
20. A set of 15 different integers have a range of 25 and a median of 25. What is greatest possible integer that could be in this set?
- 32                      37                      40                      43                      50

### Standard Deviation

1. Find the SD of 7, 8, 9 and 10.
2. Set A consists of all prime numbers between 10 and 25; Set B consists of consecutive even integers, and set C consists of consecutive multiples of 7. If all the three sets have an equal number of terms, which of the following represents the ranking of these sets in an ascending order of the standard deviation?
- (A) C, A, B      (B) A, B, C      (C) C, B, A      (D) B, C, A      (E) B, A, C

3. Set A consists of all even integers between 2 and 100, inclusive. Set X is derived by reducing each term in set A by 50, set Y is derived by multiplying each term in set A by 1.5, and set Z is derived by dividing each term in set A by -4. Which of the following represents the ranking of the three sets in descending order of standard deviation?  
 (A) X, Y, Z (B) X, Z, Y (C) Y, Z, X (D) Y, X, Z (E) Z, Y, X
4. If M is a negative integer and K is a positive integer, which of the following could be the standard deviation of a set  $\{-7, -5, -3, M, 0, 1, 3, K, 7\}$ ?  
 I. -1.5 II. -2 III. 0  
 (A) I only (B) II only (C) III only (D) I and III only (E) None
5. Sets A, B and C are shown below. If number 100 is included in each of these sets, which of the following represents the correct ordering of the sets in terms of the absolute increase in their standard deviation, from largest to smallest?  
 A  $\{30, 50, 70, 90, 110\}$ , B  $\{-20, -10, 0, 10, 20\}$ , C  $\{30, 35, 40, 45, 50\}$   
 (A) A, C, B (B) A, B, C (C) C, A, B (D) B, A, C (E) B, C, A
6. If sets X and Y have an equal number of elements, does set X have a greater standard deviation than set Y?  
 (1) The difference between each pair of the neighboring elements is consistent throughout each set;  
 (2) Each of the first two elements in Set Y is twice greater than the corresponding first and second elements in Set X.
7. The table below represents three sets of numbers with their respective medians, means and standard deviations. The third set, Set  $[A+B]$ , denotes the set that is formed by combining Set A and Set B.

	Median	Mean	Standard Deviation
Set A	X	Y	Z
Set B	L	M	N
Set $[A + B]$	Q	R	S

- If  $X - Y > 0$  and  $L - M = 0$ , then which of the following must be true?  
 I.  $Z > N$  II.  $R > M$  III.  $Q > R$   
 (A) I only (B) II only (C) III only (D) I and II only (E) None
8. If the mean of a data set is 75 and the standard deviation is 10, what is the range of scores that fall within one standard deviation of the mean?
9. The mean score of a class on a test was 60 and the standard deviation was 15. If Elena's score was within 2 standard deviations of the mean, what is the lowest score she could have received?
10. If  $y = ax + b$ , and if the standard deviation of x series is 'S', what is the standard deviation of y series?
11. If  $ax + by + c = 0$ , and if the standard deviation of x series is 'S', what is the standard deviation of y series?
12. If  $y = |x| - 100$ , and if the standard deviation of x series is 'S', what is the standard deviation of y series?
13. Three fair coins are labeled with a zero (0) on one side and a one (1) on the other side. Jimmy flips all three coins at once and computes the sum of the numbers displayed. He does this over 1000 times, writing down the sums in a long list. What is the expected standard deviation of the sums on this list?  
 (A)  $\frac{1}{2}$  (B)  $\frac{3}{4}$  (C)  $\frac{\sqrt{3}}{2}$  (D)  $\frac{\sqrt{5}}{2}$  (E)  $\frac{5}{4}$
14. Let Set  $T = \{2, 4, 5, 7\}$ . Which of the following values, if added to Set T, would most increase the standard deviation of Set T?  
 1 3 6 8 14
15. What is the standard deviation of Q, a set of consecutive integers?  
 (1) Q has 21 members.  
 (2) The median value of set Q is 20.



16. Does data set  $A = \{1, 2, x\}$  have a greater standard deviation than data set  $B = \{1, 2, 3\}$ ?  
 (1)  $x$  is greater than 3. (2)  $x$  is less than 1.
17. 9.4, 9.9, 9.9, 9.9, 10.0, 10.2, 10.2, 10.5  
 The mean and the standard deviation of the 8 numbers shown are 10 and 0.3, respectively. What percentage of the 8 numbers are within 1 standard deviation?  
 A) 90% B) 85% C) 80% D) 75% E) 70%
18. 70, 75, 80, 85, 90, 105, 105, 130, 130, 130  
 The list shown consists of the times, in seconds, that 10 schoolchildren took to run a distance of 400 meters. If the standard deviation of the 10 running times is 22.4 seconds, rounded to the nearest tenth of a second, how many of the 10 running times are more than 1 standard deviation below the mean of the 10 running times?  
 a) one b) two c) three d) four e) five
19. The residents of town  $x$  participated in a survey to determine the number of hours per week each resident spent watching television. The distribution of the result of the survey had a mean of 21 hours and a standard deviation of 6 hours. The number of hours of that participated, a resident of town  $x$  watching television last week was between 1 and 2 standard deviations below the mean. Which of the following could be the number of hours the participated watched television last week?  
 a.30 b.20 c.18 d.12 e.6
20. 7.51 8.22 7.86 8.36 8.09 7.83 8.30 8.01 7.73 8.25 7.96 8.53  
 A vending machine is designed to dispense 8 ounces of coffee into a cup. After a test that recorded the number of ounces of coffee in each of 1,000 cups dispensed by the vending machine, the 12 listed amounts, in ounces, were selected from the data. If the 1,000 recorded amounts have a mean of 8.1 ounces and a standard deviation of 0.3 ounce, how many of the 12 listed amounts are within 1.5 standard deviations of the mean?
21. A certain list of 100 data has an average of 6 and a standard deviation of  $d$ , where  $d$  is positive. Which of the following pairs of data, when added to the list, must result in a list of 102 data with standard deviation less than  $d$ ?  
 A. -6 and 0 B. 0 and 0 C. 0 and 6 D. 0 and 12 E. 6 and 6
22. The lifetime of all the batteries produced by a certain company in a year have a distribution that is symmetric about the mean  $m$ . If the distribution has a standard deviation of  $d$ , what percent of the distribution is greater than  $m+d$ ?  
 1) 68% of the distribution lies in the interval from  $m-d$  to  $m+d$ , inclusive.  
 2) 16% of the distribution is less than  $m-d$

## Quant Topic 3

### Inequalities + Absolute Value (Modulus)

1. If  $-1 < x < 0$ , which of the following must be true?  
 I.  $x^3 < x^2$       II.  $x^5 < 1 - x$       III.  $x^4 < x^2$   
 I only    I and II only    II and III only    I and III only    I, II and III
2. Is  $x > 0$ ?      (1)  $|x + 3| < 4$       (2)  $|x - 3| < 4$
3. If  $x$  and  $n$  are integers, is the sum of  $x$  and  $n$  less than zero?  
 (1)  $x + 3 < n - 1$       (2)  $-2x > 2n$
4. Is  $a > c$ ?      (1)  $b > d$       (2)  $ab^2 - b > b^2c - d$
5. If  $x$  is an integer, what is the value of  $x$ ?      (1)  $-5x > -3x + 10$       (2)  $-11x - 10 < 67$
6. If  $8x > 4 + 6x$ , what is the value of the integer  $x$ ?  
 (1)  $6 - 5x > -13$       (2)  $3 - 2x < -x + 4 < 7.2 - 2x$
7. Is  $a + b > c + d$ ?      (1)  $a > c$       (2)  $d < b$
8. If  $\sqrt{xy} = xy$ , what is the value of  $x + y$ ?      (1)  $x = -1/2$       (2)  $y$  is not equal to 0.
9. Is  $x > y$ ?      (1)  $x^2 > y$       (2)  $\sqrt{x} < y$
10. If  $6xy = x^2y + 9y$ , what is the value of  $xy$ ?      (1)  $y - x = 3$       (2)  $x^3 < 0$
11. What is the value of  $x$ ?      (1)  $x^2 - 5x + 6 = 0$       (2)  $x > 0$
12. What is  $x$ ?      (1)  $x^2 + 3x + 2 = 0$       (2)  $x < 0$
13. If  $3|3 - x| = 7$ , what is the product of all the possible values of  $x$ ?  
 $1/9$        $1/3$        $2/3$        $16/9$        $32/9$
14. Is  $a/b < 0$ ?  
 (1)  $a^2 / b^3 < 0$       (2)  $ab^4 < 0$
15. Is  $d$  negative?      (1)  $e + d = -12$       (2)  $e - d < -12$
16. If  $a - b > a + b$ , where  $a$  and  $b$  are integers, which of the following must be true?  
 I.  $a < 0$       II.  $b < 0$       III.  $ab < 0$   
 I only      II only      I and II only      I and III only      II and III only
17. If  $|a| = 1/3$  and  $|b| = 2/3$ , which of the following CANNOT be the result of  $a + b$ ?  
 $-1$        $-1/3$        $1/3$        $2/3$        $1$
18. If  $|a| = |b|$ , which of the following must be true?  
 I.  $a = b$       II.  $|a| = -b$       III.  $-a = -b$   
 I only      II only      III only      I and III only      None
19. Which of the following inequalities has a solution set that when graphed on the number line, is a single segment of finite length?  
 A.  $x^4 \geq 1$       B.  $x^3 \leq 27$       C.  $x^2 \geq 16$   
 D.  $2 \leq |x| \leq 5$       E.  $2 \leq 3x + 4 \leq 6$
20. If  $n$  is a nonzero integer, is  $x^n < 1$ ?      (1)  $x > 1$       (2)  $n > 0$
21. If  $x$  is an integer, is  $3x$  less than 500?      (1)  $4^{x-1} < 4^x - 120$       (2)  $x^2 = 36$

22. Is  $x^3 > 1$ ? (1)  $x > -2$  (2)  $2x - (b - c) < c - (b - 2)$
23. If  $\sqrt{[(x + 4)^2]} = 3$ , which of the following could be the value of  $x - 4$ ?  
-11                      -7                      -4                      -3                      5
24. Is  $x > 10^{10}$ ? (1)  $x > 2^{34}$  (2)  $x = 2^{35}$
25. Is  $XY > 0$ ? 1).  $X - Y > -2$  2).  $X - 2Y < -6$
26. If  $|x - (9/2)| = 5/2$ , and if  $y$  is the median of a set of  $p$  consecutive integers, where  $p$  is odd, which of the following must be true?  
I.  $xyp$  is odd                      II.  $xy(p^2 + p)$  is even                      III.  $x^2y^2p^2$  is even  
II only                      III only                      I and III                      II and III                      I, II, and III
27. If  $|x| + |y| = -x - y$  and  $xy$  does not equal 0, which of the following must be true?  
 $x + y > 0$                        $x + y < 0$                        $x - y > 0$                        $x - y < 0$                        $x^2 - y^2 > 0$
28. If  $x$  and  $y$  are integers and  $xy$  does not equal 0, is  $xy < 0$ ?  
(1)  $y = x^4 - x^3$                       (2)  $-12y^2 - y^2x + x^2y^2 > 0$
29. Is  $x$  a negative number?  
(1)  $x^2$  is a positive number.                      (2)  $x \cdot |y|$  is not a positive number.
30. If  $a, b, c$ , and  $d$  are integers and  $ab^2c^3d^4 > 0$ , which of the following must be positive?  
I.  $a^2cd$                       II.  $bc^4d$                       III.  $a^3c^3d^2$   
I only                      II only                      III only                      I and III                      I, II, and III
31. Is  $x|y| > y^2$ ? (1)  $x > y$  (2)  $y > 0$
32. What is  $x$ ? (1)  $|x| < 2$  (2)  $|x| = 3x - 2$
33. Is  $x > y$ ? (1)  $\sqrt{x} > y$  (2)  $x^3 > y$
34. If  $x$  is not equal to 0, is  $|x|$  less than 1? (1)  $x / |x| < x$  (2)  $|x| > x$
35. If  $r + s > 2t$ , is  $r > t$ ? (1)  $t > s$  (2)  $r > s$
36. If  $a$  and  $b$  are integers, and  $|a| > |b|$ , is  $a \cdot |b| < a - b$ ?  
(1)  $a < 0$                       (2)  $ab \geq 0$
37. Is  $a > c$ ? (1)  $b > d$  (2)  $ab^2 - b > b^2c - d$
38. If  $p < q$  and  $p < r$ , is  $(p)(q)(r) < p$ ? (1)  $pq < 0$  (2)  $pr < 0$
39. If  $|x| \cdot y + 9 > 0$ , and  $x$  and  $y$  are integers, is  $x < 6$ ? (1)  $y$  is negative (2)  $|y| \leq 1$
40. If  $n$  is not equal to 0, is  $|n| < 4$ ? (1)  $n^2 > 16$  (2)  $1/|n| > n$
41. If  $x$  and  $y$  are non-zero integers and  $|x| + |y| = 32$ , what is  $xy$ ?  
(1)  $-4x - 12y = 0$                       (2)  $|x| - |y| = 16$
42. What is the value of  $y$ ? (1)  $3|x^2 - 4| = y - 2$  (2)  $|3 - y| = 11$
43. Is  $x > 0$ ? (1)  $|x + 3| = 4x - 3$  (2)  $|x - 3| = |2x - 3|$
44. What is the value of  $|x|$ ? (1)  $|x^2 + 16| - 5 = 27$  (2)  $x^2 = 8x - 16$
45. If  $x > y$ ,  $x^2 - 2xy + y^2 - 9 = 0$ , and  $x + y = 15$ , what is  $x$ ?  
3                      6                      12                      18                      9
46. Is  $|n| < 1$ ? (1)  $n^x - n < 0$  (2)  $x^{-1} = -2$
47. Is  $5^n < 0.04$ ? (1)  $(1/5)^n > 25$  (2)  $n^3 < n^2$

48. What is the ratio of  $2x$  to  $3y$ ?  
 (1) The ratio of  $x^2$  to  $y^2$  is equal to  $36/25$ . (2) The ratio of  $x^5$  to  $y^5$  is greater than 1.
49. If  $x$  and  $y$  are integers, does  $x^y y^{-x} = 1$ ? (1)  $x^x > y$  (2)  $x > y^y$
50. If  $a$  is nonnegative, is  $x^2 + y^2 > 4a$ ? (1)  $(x + y)^2 = 9a$  (2)  $(x - y)^2 = a$
51. If  $k$  is a positive constant and  $y = |x - k| - |x + k|$ , what is the maximum value of  $y$ ?  
 (1)  $x < 0$  (2)  $k = 3$
52. If  $x > 0$ , what is the least possible value for  $x + (1/x)$ ?  
 (A) 0.5 (B) 1 (C) 1.5 (D) 2 (E) 2.5
53. Is  $(|x^{-1}y^{-1}|)^{-1} > xy$ ? (1)  $xy > 1$  (2)  $x^2 > y^2$
54. Is  $xy + xy < xy$ ? (1)  $x^2 / y < 0$  (2)  $x^9 (y^3)^3 < (x^2)^4 y^8$
55.  $w, x, y$ , and  $z$  are positive integers. If  $w/x < y/z < 1$ , what is the proper order of magnitude, increasing from left to right, of the following quantities:  
 $x/w, z/y, x^2/w^2, xz/wy, (x + z) / (w + y), 1$ ?
56. Two missiles are launched simultaneously. Missile 1 launches at a speed of  $x$  miles per hour, increasing its speed by a factor of  $\sqrt{x}$  every 10 minutes (so that after 10 minutes its speed is  $x\sqrt{x}$ , after 20 minutes its speed is  $x^2$ , and so forth). Missile 2 launches at a speed of  $y$  miles per hour, doubling its speed every 10 minutes. After 1 hour, is the speed of Missile 1 greater than that of Missile 2?  
 1)  $x = \sqrt{y}$  2)  $x > 8$
57.  $8xy^3 + 8x^3y = 2x^2y^2 / 2^{-3}$ , What is  $xy$ ?  
 (1)  $y > x$  (2)  $x < 0$
58. If  $(a - b)c < 0$ , which of the following cannot be true?  
 $a < b$   $c < 0$   $|c| < 1$   $ac > bc$   $a^2 - b^2 > 0$
59. If  $|ab| > ab$ , which of the following must be true?  
 I.  $a < 0$  II.  $b < 0$  III.  $ab < 0$   
 I only II only III only I and III II and III
60. If  $b < c < d$  and  $c > 0$ , which of the following cannot be true if  $b, c$  and  $d$  are integers?  
 $bcd > 0$   $b + cd < 0$   $b - cd > 0$   $b/cd < 0$   $b^3cd < 0$
61. If  $ab > cd$  and  $a, b, c$  and  $d$  are all greater than zero, which of the following CANNOT be true?  
 $c > b$   $d > a$   $b/c > d/a$   $a/c > d/b$   $(cd)^2 < (ab)^2$
62. Is  $x + y > 0$ ?  
 (1)  $x - y > 0$  (2)  $x^2 - y^2 > 0$
63. Is  $|x| < 1$ ?  
 (1)  $|x + 1| = 2|x - 1|$  (2)  $|x - 3| > 0$
64. Is  $|a| > |b|$ ?  
 (1)  $b < -a$  (2)  $a < 0$
65. If  $r$  is not equal to 0, is  $r^2 / |r| < 1$ ?  
 (1)  $r > -1$  (2)  $r < 1$
66. Which of the following sets includes ALL of the solutions of  $x$  that will satisfy the equation:  $|x - 2| - |x - 3| = |x - 5|$ ?  
 $\{-6, -5, 0, 1, 7, 8\}$   $\{-4, -2, 0, 10/3, 4, 5\}$   
 $\{-4, 0, 1, 4, 5, 6\}$   $\{-1, 10/3, 3, 5, 6, 8\}$   
 $\{-2, -1, 1, 3, 4, 5\}$

67. If  $abc \neq 0$ , what is the value of  $(a^3 + b^3 + c^3) / abc$ ?  
 (1)  $|a|=1, |b|=2, |c|=3$  (2)  $a + b + c = 0$
68. Given that  $w = |x|$  and  $x = 2^b - (8^{30} + 8^5)$ , which of the following values for  $b$  yields the lowest value for  $w$ ?  
 (A) 35 (B) 90 (C) 91 (D) 95 (E) 105
69. If  $x$  is an integer, what is the value of  $x$ ?  
 1)  $|x - |x^2|| = 2$  2)  $|x^2 - |x|| = 2$
70.  $w, x, y$ , and  $z$  are integers. If  $z > y > x > w$ , is  $|w| > x^2 > |y| > z^2$ ?  
 1)  $wx > yz$  2)  $zx > wy$
71. If  $ab \neq 0$ , is  $\left(\frac{a-b}{a-1-b-1}\right)^{-1} > a + b$ ?  
 (1)  $|a| > |b|$  (2)  $a < b$
72. Is  $|a| + |b| > |a + b|$ ?  
 (1)  $a^2 > b^2$  (2)  $|a| \times b < 0$
73. Is  $\sqrt{x}$  a prime number?  
 (1)  $|3x - 7| = 2x + 2$  (2)  $x^2 = 9x$
74. What is the average of  $x$  and  $|y|$ ?  
 (1)  $x + y = 20$  (2)  $|x + y| = 20$
75. If  $x$  and  $y$  are nonzero integers, is  $(x^{-1} + y^{-1})^{-1} > [(x^{-1})(y^{-1})]^{-1}$ ?  
 (1)  $x = 2y$  (2)  $x + y > 0$
76. Is  $p^2q > pq^2$ ? (1)  $pq < 0$   $p < 0$
77. Is  $m > n$ ? (1)  $n - m + 2 > 0$  (2)  $n - m - 2 > 0$
78. Is  $3^p > 2^q$ ? (1)  $q = 2p$  (2)  $q > 0$
79. Is  $mp$  greater than  $m$ ? (1)  $m > p > 0$  (2)  $p$  is less than 1
80. Is  $w$  less than  $y$ ?  
 (1)  $1.3 < w < 1.3101$  (2)  $1.3033 < y$
81. If  $a$  and  $b$  are integers and  $a \neq b$ , is  $|a|b > 0$ ?  
 (1)  $|a^b| > 0$  (2)  $|a|^b$  is a non-zero integer
82. If 500 is the multiple of 100 that is closest to  $X$  and 400 is the multiple of 100 that is closest to  $Y$ , which multiple of 100 is closest to  $X+Y$ ?  
 1).  $X < 500$  2).  $Y < 400$
83. Is the three-digit number  $n$  less than 550?  
 1). the product of the digits in  $n$  is 30 2). the sum of the digits in  $n$  is 10
84. If  $X^4 + Y^4 = 100$ , then the greatest possible value of  $X$  is between:  
 A. 0 and 3 B. 3 and 6 C. 6 and 9 D. 9 and 12 E. 12 and 15
85. Is  $2X - 3Y < X^2$ ? 1).  $2X - 3Y = -2$  2).  $X > 2$  and  $Y > 0$
86. Is  $m + z > 0$  1).  $m - 3z > 0$  2).  $4z - m > 0$
87. If  $X > Y^2 > Z^4$ , which of the following statements could be true?  
 I.  $X > Y > Z$  II.  $Z > Y > X$  III.  $X > Z > Y$   
 A. I only B. I and II only C. I and III only  
 D. II and III only E. I, II, and III
88. Is  $X + Y < 1$  1).  $x < 8/9$  2).  $Y < 1/8$

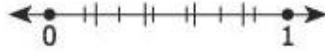
89. If  $y$  is an integer and  $y = x + |x|$ , is  $y = 0$ ? 1).  $x < 0$  2).  $y < 1$
90. Is  $x - y + 1$  greater than  $x + y - 1$ ? 1).  $x > 0$  2).  $y < 0$
91. Is  $W$  greater than 1? 1).  $W + 2 > 0$  2).  $W^2 > 1$
92. If  $n$  and  $p$  are integers, is  $p > 0$ ? 1).  $n + 1 > 0$  2).  $np > 0$
93. The number  $x$  and  $y$  are not integers, the value of  $x$  is closest to which integer?  
1). 4 is the integer that is closest to  $x + y$   
2). 1 is the integer that is closest to  $x - y$

## GMAT Quant Topic 4 (Numbers)

### Types of numbers

- What is the sum of the digits of the positive integer  $n$  where  $n < 99$ ?  
(1)  $n$  is divisible by the square of the prime number  $y$ .  
(2)  $y^4$  is a two-digit odd integer.
- If  $x$  is a positive integer, is  $x! + (x + 1)$  a prime number?  
(1)  $x < 10$  (2)  $x$  is even
- Is  $\sqrt{(x + y)}$  an integer?  
(1)  $x^3 = 64$  (2)  $x^2 = y - 3$
- If  $x$  is a prime number, what is the value of  $x$ ?  
(1)  $2x + 2$  is the cube of a positive integer.  
(2) The average of any  $x$  **consecutive integers** is an integer.
- List K consists of 12 **consecutive integers**, if -4 is the least integer in list K, what is the range of the positive integers in the list K?
- If  $m$ ,  $r$ ,  $x$  and  $y$  are positive, is the ratio of the  $m$  to  $r$  equal to the ratio of  $x$  to  $y$ ?  
1) the ratio of  $m$  to  $y$  is equal to the ratio of  $x$  to  $r$   
2) the ratio of  $m+x$  to  $r+y$  is equal to the ratio of  $x$  to  $y$
- If the integer  $a$  and  $n$  are greater than 1, and the product of the first 8 positive integers is a multiple of  $a^n$ , what is the value of  $a$ ?  
1).  $a^n = 64$  2).  $n = 6$
- If  $x$  is the sum of six **consecutive integers**, then  $x$  is divisible by which of the following:  
I. 3 II. 4 III. 6  
I only II only III only I and III I, II, and III
- In a certain deck of cards, each card has a positive integer written on it. In a multiplication game, a child draws a card and multiplies the integer on the card by the next larger integer. If each possible product is between 15 and 200, then the least and greatest integers on the cards could be  
3 and 15 3 and 20 4 and 13 4 and 14
- If  $p$  is a positive integer, what is the value of  $p$ ?  
1).  $p/4$  is a prime number 2).  $p$  is divisible by 3
- The number 75 can be written as the sum of the squares of 3 different positive integers. What is the sum of these 3 integers?  
17 16 15 14 13
- An integer greater than 1 that is not prime is called composite. If the two-digit integer  $n$  is greater than 20, is  $n$  composite?  
1). the tens digit of  $n$  is a factor of the units digit of  $n$   
2). the tens digit of  $n$  is 2.
- If  $n$  is a multiple of 5 and  $n = p^2q$ , where  $p$  and  $q$  are prime numbers, which of the following must be a multiple of 25?  
 $p^2$   $q^2$   $pq$   $p^2q^2$   $p^3q$
- On the number line shown, is zero halfway between  $r$  and  $s$ ?  
----r---- s---- t---  
1).  $s$  is to the right of zero  
2). the distance between  $t$  and  $r$  is the same as the distance between  $t$  and  $-s$ .
- What is the sum of the first 10 prime numbers?  
100 101 128 129 158

16. On the number line, the segment from 0 to 1 has been divided into fifths, as indicated by the large tick marks, and also into sevenths, as indicated by the small tick marks. What is the least possible distance between any two of the tick marks?



17. For non-zero integers  $a$ ,  $b$ ,  $c$  and  $d$ , is  $ab/cd$  positive?  
 (1)  $ad + bc = 0$  (2)  $abcd = -4$
18. Is the positive integer  $J$  divisible by a greater number of different prime numbers than the positive integer  $k$ ?  
 1).  $J$  is divisible by 30 2).  $k=1000$
19. If  $n$  is a positive integer and the product of all the integers from 1 to  $n$ , inclusive, is a multiple of 990, what is the least possible value of  $n$ ?
20. For which of the following functions is  $f(a+b)=f(b)+f(a)$  for all positive numbers  $a$  and  $b$ ?  
 A.  $f(x)=x^2$  B.  $f(x)=x+1$  C.  $f(x)=\sqrt{x}$  D.  $f(x)=2/x$  E.  $f(x)=-3x$
21. The point  $A$ ,  $B$ ,  $C$ , and  $D$  are on the number line, not necessarily in the order. If the distance between  $A$  and  $B$  is 18 and the distance between  $C$  and  $D$  is 8, what is the distance between  $B$  and  $D$ ?  
 1). The distance between  $C$  and  $A$  is the same as the distance between  $C$  and  $B$ .  
 2).  $A$  is to the left of  $D$  on the number line.
22. A certain list consists of several different integers. Is the product of all the integers in the list positive?  
 1). the product of the greatest and the smallest of the integers in the list are positive.  
 2). There is even number of integers in the list.
23. The sum of positive integers  $x$  and  $y$  is 77. What is the value of  $xy$ ?  
 1).  $x=y+1$   
 2).  $x$  and  $y$  have the same tens' digit.
24. If there are more than two numbers in certain list, is each of the numbers in the list equal to 0?  
 1). The product of any two numbers in the list equal to 0.  
 2). The sum of any two numbers in the list equal to 0.
25. For which of the following values of  $x$  is  $\{1-[2-(x^{1/2})]^{1/2}\}^{1/2}$  not defined as a real number?  
 1 2 3 4 5
26. For a finite sequence of nonzero numbers, the number of variations in sign is defined as the number of pairs of consecutive terms of the sequence for which the product of the two consecutive terms is negative. What is the number of variations in sign for the sequence: 1, -3, 2, 5, -4, -6?
27. If  $xy + z = x(y+z)$ , which of the following must be true?  
 $x=0$  and  $z=0$        $x=1$  and  $y=1$        $y=1$  and  $z=0$        $x=1$  or  $y=0$   
 $x=1$  or  $z=0$
28. Symbol  $*$  denote to be one of the operations add, subtract, multiply, or divide. Is  $(6*2)*4=6*(2*4)$ ?  
 1).  $3*2>3$  2).  $3*1=3$
29. If  $m$  and  $r$  are two numbers on a number line, what is the value of  $r$ ?  
 1). The distance between  $r$  and 0 is 3 time the distance between  $m$  and 0.  
 2). 12 is halfway between  $m$  and  $r$
30. As the table shows,  $m+n=?$

+	x	Y	z
4	1	-5	m
E	7	N	10
F	2	-4	5

31. If  $w$ ,  $y$ , and  $z$  are positive integers, and  $w = y - z$ , is  $w$  a perfect square?  
 (1)  $y + z$  is a perfect square. (2)  $z$  is even.



## Odds and Evens

1. Is  $z$  even? (1)  $z/2$  is even. (2)  $3z$  is even.
2. If  $m$ ,  $n$ , and  $p$  are integers, is  $m + n$  odd?  
(1)  $m = p^2 + 4p + 4$  (2)  $n = p^2 + 2m + 1$
3. If  $a$  and  $b$  are both positive integers, is  $b^{a+1} - ba^b$  odd?  
(1)  $a + (a + 4) + (a - 8) + (a + 6) + (a - 10)$  is odd  
(2)  $b^3 + 3b^2 + 5b + 7$  is odd
4. Is the positive integer  $x$  odd?  
(1)  $x = y^2 + 4y + 6$ , where  $y$  is a positive integer.  
(2)  $x = 9z^2 + 7z - 10$ , where  $z$  is a positive integer.
5. If  $w$ ,  $y$ , and  $z$  are positive integers, and  $w = y - z$ , is  $w$  a perfect square?  
(1)  $y + z$  is a perfect square. (2)  $z$  is even.
6. If  $x$  and  $y$  are positive integers and  $3x + 5 < x + 11$ , is  $x$  a prime number?  
(1) The sum of  $x$  and  $y$  is even. (2) The product of  $x$  and  $y$  is odd.
7. Is the positive integer  $p$  even? (1)  $p^2 + p$  is even. (2)  $4p + 2$  is even.
8. If  $p$  and  $q$  are integers and  $p + q + p$  is odd, which of the following must be odd?  

$p$	$q$	$p + q$	$pq$	$pq + p$
-----	-----	---------	------	----------
9. If  $a$ ,  $b$ , and  $c$  are integers and  $ab^2 / c$  is a positive even integer, which of the following must be true?  

I. $ab$ is even	II. $ab > 0$	III. $c$ is even
I only	II only	I and II
		I and III
		I, II, and III
10. If  $k$  and  $y$  are integers, and  $10k + y$  is odd, which of the following must be true?  

$k$ is odd	$k$ is even	$y$ is odd
$y$ is even	both $k$ and $y$ are odd	
11. Each digit in the two-digit number  $G$  is halved to form a new two-digit number  $H$ . Which of the following could be the sum of  $G$  and  $H$ ?  

153	150	137	129	89
-----	-----	-----	-----	----
12. If  $a$  is an even integer and  $b$  is an odd integer, which of the following cannot be an even integer?  

$ab$	$a/b$	$b/a$	$a^b$	$a^{2b+1}$
------	-------	-------	-------	------------
13. If  $x$  and  $y$  are prime integers and  $x < y$ , which of the following cannot be true?  

$x$ is even	$x + y$ is odd	$xy$ is even	$y + xy$ is even	$2x + y$ is even
-------------	----------------	--------------	------------------	------------------
14. If  $q$ ,  $r$ , and  $s$  are **consecutive even integers** and  $q < r < s$ , which of the following CANNOT be the value of  $s^2 - r^2 - q^2$ ?  
(A) -20 (B) 0 (C) 8 (D) 12 (E) 16
15.  $n$  is an integer greater than or equal to 0. The sequence  $t_n$  for  $n > 0$  is defined as  $t_n = t_{n-1} + n$ . Given that  $t_0 = 3$ , is  $t_n$  even?  
(1)  $n + 1$  is divisible by 3 (2)  $n - 1$  is divisible by 4
16.  $y$  and  $z$  are nonzero integers, is the square of  $(y + z)$  even?  
(1)  $y - z$  is odd. (2)  $yz$  is even.
17. If  $x$  and  $y$  are positive integers, is the product  $xy$  even?  
1).  $5x - 4y$  is even 2).  $6x + 7y$  is even.
18. If  $x$  and  $y$  are integers, is  $x(y+1)$  an even number?  
1).  $x$ , and  $y$  are prime numbers. 2).  $y > 7$

19. For all positive integers  $m$ ,  $(m) = 3m$  when  $m$  is odd and  $(m) = \frac{1}{2}m$  when  $m$  is even, which of the following is equivalent to  $(9)*(6)$ ?  
 (81) (54) (36) (27) (18)
20. If  $m$  and  $n$  are integers, is  $m$  odd?  
 1).  $m+n$  is odd 2).  $m+n = n^2 + 5$
21. If  $c$  and  $d$  are integers, is  $C$  even?  
 1).  $c(d+1)$  is even 2).  $(c+2)(d+4)$  is even
22. If  $x$  is an integer, is  $(x^2+1)(x+5)$  an even number?  
 1).  $x$  is an odd number. 2). each prime factor of  $x^2$  is greater than 7
23. If  $a$  is an even integer and  $b$  is an odd integer, which of the following cannot be an even integer?  
 $ab$   $a/b$   $b/a$   $a^b$   $a^{2b+1}$
24. If  $y$  and  $z$  are nonzero integers, is the square of  $(y+z)$  even?  
 (1)  $y-z$  is odd. (2)  $yz$  is even.
25. If  $x$  and  $y$  are prime integers and  $x < y$ , which of the following cannot be true?  
 $x$  is even  $x+y$  is odd  $xy$  is even  
 $y+xy$  is even  $2x+y$  is even

### Unit's digits, factorial powers

1.  $17^{27}$  has a units digit of:  
 1 2 3 7 9
2. If  $r$ ,  $s$ , and  $t$  are all positive integers, what is the remainder of  $2^p / 10$ , if  $p = rst$ ?  
 (1)  $s$  is even (2)  $p = 4t$
3.  $1^1+2^2+3^3+\dots+10^{10}$  is divided by 5. What is the remainder?  
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. Given that  $p$  is a positive even integer with a positive units digit, if the units digit of  $p^3$  minus the units digit of  $p^2$  is equal to 0, what is the units digit of  $p+3$ ?  
 3 6 7 9 It cannot be determined from the given information.
5. If  $x$  is a positive integer, what is the units digit of  $(24)^{(2x+1)}(33)^{(x+1)}(17)^{(x+2)}(9)^{(2x)}$ ?  
 (A) 4 (B) 6 (C) 7 (D) 8 (E) 9
6. If  $a$  and  $b$  are positive integers and  $x = 4^a$  and  $y = 9^b$ , which of the following is a possible units digit of  $xy$ ?  
 1 4 5 7 8
7. If  $x = 3^{21}$  and  $y = 6^{55}$ , what is the remainder when  $xy$  is divided by 10?  
 (A) 2 (B) 3 (C) 4 (D) 6 (E) 8
8. If  $x$  is a positive integer, what is the remainder when  $7^{12x+3} + 3$  is divided by 5?  
 0 1 2 3 4
9. If  $x$  and  $y$  are positive integers and  $n = 5^x + 7^{y+15}$ , what is the units digit of  $n$ ?  
 (1)  $y = 2x - 15$  (2)  $y^2 - 6y + 5 = 0$
10. What is the units digit of  $(71)^5(46)^3(103)^4 + (57)(1088)^3$ ?  
 0 1 2 3 4
11. If  $\frac{(13!)^{16} - (13!)^8}{(13!)^8 + (13!)^4} = a$ , what is the units digit of  $\frac{a}{(13!)^4}$ ?  
 (A) 0 (B) 1 (C) 3 (D) 5 (E) 9

12. What is the units digit of  $177^{28} - 133^{23}$ ?  
 (A) 1 (B) 3 (C) 4 (D) 6 (E) 9
13. What is the greatest integer  $m$  for which the number  $50! / 10^m$  is an integer?  
 (A) 5 (B) 8 (C) 10 (D) 11 (E) 12
14. How many terminating zeroes does  $200!$  have?  
 (A) 40 (B) 48 (C) 49 (D) 55 (E) 64
15. If  $(243)^x(463)^y = n$ , where  $x$  and  $y$  are positive integers, what is the units digit of  $n$ ?  
 (1)  $x + y = 7$  (2)  $x = 4$
16. If  $y$  is divisible by the square of an even prime number and  $x$  is the actual square of an even prime number, then what is the units digit of  $x^y$ ?  
 0 2 4 6 8
17. If  $x$  is a positive integer, what is the units digit of  $x^2$ ?  
 (1) The units digit of  $x^4$  is 1. (2) The units digit of  $x$  is 3.

### Decimals

1. In the number  $1.4ab5$ ,  $a$  and  $b$  represent single positive digits. If  $x = 1.4ab5$ , what is the value of  $10 - x$ ?  
 (1) If  $x$  is rounded to the nearest hundredth, then  $10 - x = 8.56$ .  
 (2) If  $x$  is rounded to the nearest thousandth, then  $10 - x = 8.564$ .
2. If  $a, b, c, d$  and  $e$  are integers and  $p = 2^a 3^b$  and  $q = 2^c 3^d 5^e$ , is  $p/q$  a terminating decimal?  
 (1)  $a > c$  (2)  $b > d$
3. If the fraction  $d$  were converted into a decimal, would there be more than 3 nonzero digits to the right of the decimal point?  
 (1) The denominator of  $d$  is exactly 8 times the numerator of  $d$ .  
 (2) If  $d$  were converted into a decimal,  $d$  would be a non-repeating decimal.
4. If  $x$  is an integer, can the number  $(5/28)(3.02)(90\%)(x)$  be represented by a finite number of non-zero decimal digits?  
 (1)  $x$  is greater than 100 (2)  $x$  is divisible by 21
5. Given that  $a, b, c$ , and  $d$  are non-negative integers, is the fraction  $(ad) / (2^a 3^b 4^c 5^d)$  a terminating decimal?  
 (1)  $d = (1 + a)(a^2 - 2a + 1) / (a - 1)(a^2 - 1)$  (2)  $b = (1 + a)(a^2 - 2a + 1) - (a - 1)(a^2 - 1)$
6. If  $d$  represents the hundredths digit and  $e$  represents the thousandths digit in the decimal  $0.4de$ , what is the value of this decimal rounded to the nearest tenth?  
 (1)  $d - e$  is equal to a positive perfect square. (2)  $\sqrt{d} > e^2$
7. Is the hundredth digit of decimal  $d$  greater than 5?  
 1). The tenth digit of  $10d$  is 7 2). The thousandth digit of  $d/10$  is 7
8. The value of  $x$  is derived by summing  $a, b$ , and  $c$  and then rounding the result to the tenths place. The value of  $y$  is derived by first rounding  $a, b$ , and  $c$  to the tenths place and then summing the resulting values. If  $a = 5.45$ ,  $b = 2.98$ , and  $c = 3.76$ , what is  $y - x$ ?  
 -.1 0 .05 .1 .2
9. What is the value of the tenths digit of number  $x$ ?  
 (1) The hundredths digit of  $x$  is 5  
 (2) Number  $x$ , rounded to the nearest tenth, is 54.5
10. If  $x$  and  $y$  each represent a single digit, does the number  $8.3xy$  round to 8.3 when it is rounded to the nearest tenth?  
 (1)  $x = 5$  (2)  $y = 9$
11. If  $j$  and  $k$  each represent positive single digits, and  $y = 2.\dot{3}k$ , what is  $y$  rounded to the nearest tenth?  
 (1)  $j > k$  (2) If  $y$  is rounded to the nearest hundredth, the result is 2.74.

12. If the fraction  $d$  were converted into a decimal, would there be more than 3 nonzero digits to the right of the decimal point?  
 (1) The denominator of  $d$  is exactly 8 times the numerator of  $d$ .  
 (2) If  $d$  were converted into a decimal,  $d$  would be a non-repeating decimal.

**13.  $d = 83,521,y73/441,682,36y$**

In the expression above, the letter  $y$  represents a single digit from 0 to 9. Is  $d$  a decimal with exactly ten digits?

- (1) The sum of all the digits in the numerator is not a multiple of 3.  
 (2) 33 is a factor of the denominator.

**Sequences and Series**

1. If integer  $k$  is equal to the sum of all even multiples of 15 between 295 and 615, what is the greatest prime factor of  $k$ ?  
 5                      7                      11                      13                      17
2. If  $S$  is the infinite sequence  $S_1 = 6, S_2 = 12, \dots, S_n = S_{n-1} + 6, \dots$ , what is the sum of all terms in the set  $\{S_{13}, S_{14}, \dots, S_{28}\}$ ?  
 1,800                      1,845                      1,890                      1,968                      2,016
3. In an increasing sequence of 5 consecutive even integers, the sum of the second, third, and fourth integer is 132. What is the sum of the first and last integers?  
 84                      86                      88                      90                      92
4. What is the sum of the multiples of 7 from 84 to 140, inclusive?  
 896    963    1008    1792    2016
5. In a sequence of terms in which each term is three times the previous term, what is the fourth term?  
 (1) The first term is 3.                      (2) The second to last term is  $3^{10}$ .
6. If each term in the sum  $a_1 + a_2 + a_3 + \dots + a_n$  is either 7 or 77 and the sum is equal to 350, which of the following could equal to  $n$ ?  
 38 39 40 41 42
7.  $2+2+2^2+2^3+2^4+2^5+2^6+2^7+2^8=?$
8. For any integer  $k$  from 1 to 10, inclusive, the  $k$ th of a certain sequence is given by  $[(-1)^{(k+1)}] \times (1/2^k)$ . If  $T$  is the sum of the first 10 terms of the sequence, then  $T$  is:  
 A. greater than 2                      B. between 1 and 2                      C. between  $1/2$  and 1  
 D. between  $1/4$  and  $1/2$                       E. less than  $1/4$
9. Sequence  $A$  is defined by the equation  $A_n = 3n + 7$ , where  $n$  is an integer greater than or equal to 1. If set  $B$  is comprised of the first  $x$  terms of sequence  $A$ , what is the median of set  $B$ ?  
 (1) The sum of the terms in set  $B$  is 275.  
 (2) The range of the terms in set  $B$  is 30
10.  $S$  is the infinite sequence  $S_1 = 2, S_2 = 22, S_3 = 222, \dots, S_k = S_{k-1} + 2(10^{k-1})$ . If  $p$  is the sum of the first 30 terms of  $S$ , what is the eleventh digit of  $p$ , counting right to left from the units digit?  
 1                      2                      4                      6                      9
11. Sequence  $S$  is defined as  $S_n = 2S_{n-1} - 2$ . If  $S_1 = 3$ , then  $S_{10} - S_9 =$   
 2                      120                      128                      250                      256
12.  $S_n = 2S_{n-1} + 4$  and  $Q_n = 4Q_{n-1} + 8$  for all  $n > 1$ . If  $S_5 = Q_4$  and  $S_7 = 316$ , what is the first value of  $n$  for which  $Q_n$  is an integer?  
 (A) 1                      (B) 2                      (C) 3                      (D) 4                      (E) 5
13. What is the sixtieth term in the following sequence?    1, 2, 4, 7, 11, 16, 22...  
 (A) 1,671                      (B) 1,760                      (C) 1,761                      (D) 1,771                      (E) 1,821

14. Sequence  $S$  is defined as  $S_n = X + (1/X)$ , where  $X = S_{n-1} + 1$ , for all  $n > 1$ . If  $S_1 = 201$ , then which of the following must be true of  $Q$ , the sum of the first 50 terms of  $S$ ?  
 (A)  $13,000 < Q < 14,000$  (B)  $12,000 < Q < 13,000$  (C)  $11,000 < Q < 12,000$   
 (D)  $10,000 < Q < 11,000$  (E)  $9,000 < Q < 10,000$
15. In a certain sequence, every term after the first is determined by multiplying the previous term by an integer constant greater than 1. If the fifth term of the sequence is less than 1000, what is the maximum number of nonnegative integer values possible for the first term?  
 (A) 60 (B) 61 (C) 62 (D) 63 (E) 64
16. The sum of the squares of the first 15 positive integers ( $1^2 + 2^2 + 3^2 + \dots + 15^2$ ) is equal to 1240. What is the sum of the squares of the second 15 positive integers ( $16^2 + 17^2 + 18^2 + \dots + 30^2$ )?  
 (A) 2480 (B) 3490 (C) 6785 (D) 8215 (E) 9255
17. Given a series of  $n$  consecutive positive integers, where  $n > 1$ , is the average value of this series an integer divisible by 3?  
 (1)  $n$  is odd (2) The sum of the first number of the series and  $(n - 1) / 2$  is an integer divisible by 3
18. A certain series is defined by the following recursive rule:  $S_n = k(S_{n-1})$ , where  $k$  is a constant. If the 1st term of this series is 64 and the 25th term is 192, what is the 9th term?
19. The infinite sequence  $S_k$  is defined as  $S_k = 10 S_{k-1} + k$ , for all  $k > 1$ . The infinite sequence  $A_n$  is defined as  $A_n = 10 A_{n-1} + (A_1 - (n - 1))$ , for all  $n > 1$ .  $q$  is the sum of  $S_k$  and  $A_n$ . If  $S_1 = 1$  and  $A_1 = 9$ , and if  $A_n$  is positive, what is the maximum value of  $k + n$  when the sum of the digits of  $q$  is equal to 9?  
 (A) 6 (B) 9 (C) 12 (D) 16 (E) 18
20. A certain club has exactly 5 new members at the end of its first week. Every subsequent week, each of the previous week's new members (and only these members) brings exactly  $x$  new members into the club. If  $y$  is the number of new members brought into the club during the twelfth week, which of the following could be  $y$ ?  
 (A)  $12\sqrt[5]{5}$  (B)  $3^{11}5^{11}$  (C)  $3^{12}5^{12}$  (D)  $3^{11}5^{12}$  (E)  $60^{12}$
21.  $36^2 + 37^2 + 38^2 + 39^2 + 40^2 + 41^2 + 42^2 + 43^2 + 44^2 =$   
 (A) 14400 (B) 14440 (C) 14460 (D) 14500 (E) 14520
22. A certain established organization has exactly 4096 members. A certain new organization has exactly 4 members. Every 5 months the membership of the established organization increases by 100 percent. Every 10 months the membership of the new organization increases by 700 percent. New members join the organizations only on the last day of each 5- or 10-month period. Assuming that no member leaves the organizations, after how many months will the two groups have exactly the same number of members?  
 (A) 20 (B) 40 (C) 50 (D) 80 (E) 100
23. In the infinite sequence  $A$ ,  $A_n = x^{n-1} + x^n + x^{n+1} + x^{n+2} + x^{n+3}$ , where  $x$  is a positive integer constant. For what value of  $n$  is the ratio of  $A_n$  to  $x(1 + x(1 + x(1 + x(1 + x))))$  equal to  $x^5$ ?  
 (A) 8 (B) 7 (C) 6 (D) 5 (E) 4
24. If the expression  $\sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \dots}}}}}$  extends to an infinite number of roots and converges to a positive number  $x$ , what is  $x$ ?
25. What is the sum of the even integers between 200 and 400, inclusive?  
 29,700      30,000      30,300      60,000      60,300

26.

98	-200	310	-396	498
102	-202	290	-402	502
101	-198	305	-398	501
100	-204	295	-404	500
99	-196	300	-400	499

What is the sum of all of the integers in the chart above?

0                      300                      500                      1,500                      6,500

27. The sequence  $f(n) = (2n)! \div n!$  is defined for all positive integer values of  $n$ . If  $x$  is defined as the product of the first 10 terms of this sequence, which of the following is the greatest factor of  $x$ ?
- (A)  $2^{20}$                       (B)  $2^{30}$                       (C)  $2^{45}$                       (D)  $2^{52}$                       (E)  $2^{55}$

### Remainders, Divisibility

- When the positive integer  $x$  is divided by 9, the remainder is 5. What is the remainder when  $3x$  is divided by 9?  
0                      1                      3                      4                      6
- If  $(x \# y)$  represents the remainder that results when the positive integer  $x$  is divided by the positive integer  $y$ , what is the sum of all the possible values of  $y$  such that  $(16 \# y) = 1$ ?  
8                      9                      16                      23                      24
- If  $k$  and  $x$  are positive integers and  $x$  is divisible by 6, which of the following CANNOT be the value of  $\sqrt{288k}$ ?  
 $24k\sqrt{3}$                        $24\sqrt{k}$                        $24\sqrt{(3k)}$                        $24\sqrt{(6k)}$                        $72\sqrt{k}$
- $10^{25} - 560$  is divisible by all of the following EXCEPT:  
11                      8                      5                      4                      3
- $x$ ,  $y$ ,  $a$ , and  $b$  are positive integers. When  $x$  is divided by  $y$ , the remainder is 6. When  $a$  is divided by  $b$ , the remainder is 9. Which of the following is NOT a possible value for  $y + b$ ?  
24                      21                      20                      17                      15
- In order to play a certain game, 24 players must be split into  $n$  teams, with each team having an equal number of players. If there are more than two teams, and if each team has more than two players, how many teams are there?  
(1) If thirteen new players join the game, one must sit out so that the rest can be split up evenly among the teams.  
(2) If seven new players join the game, one must sit out so that the rest can be split up evenly among the teams.
- When the positive integer  $x$  is divided by 4, is the remainder equal to 3?  
(1) When  $x/3$  is divided by 2, the remainder is 1.                      (2)  $x$  is divisible by 5.
- Seven integers,  $x_1, x_2, x_3, x_4, x_5, x_6$ , and  $x_7$ , are picked at random from the set of all integers between 10 and 110, inclusive. If each of these integers is divided by 7 and the 7 remainders are all added together, what would be the sum of the 7 remainders?  
(1) The range of the remainders is 6.                      (2) The seven integers are **consecutive**.
- When the integer  $x$  is divided by the integer  $y$ , the remainder is 60. Which of the following is a possible value of the quotient  $x/y$ ?  
I. 15.15                      II. 18.16                      III. 17.17  
(A) I only                      (B) II only                      (C) III only                      (D) I and II only  
(E) I and III only
- If  $j$  and  $k$  are positive integers where  $k > j$ , what is the value of the remainder when  $k$  is divided by  $j$ ?  
(1) There exists a positive integer  $m$  such that  $k = jm + 5$ .                      (2)  $j > 5$
- Five **consecutive positive integers** are chosen at random. If the average of the five integers is odd, what is the remainder when the largest of the five integers is divided by 4?  
(1) The third of the five integers is a prime number.  
(2) The second of the five integers is the square of an integer.
- Can a batch of identical cookies be split evenly between Laurel and Jean without leftovers and without breaking a cookie?

- (1) If the batch of cookies were split among Laurel, Jean and Marc, there would be one cookie left over.  
 (2) If Peter eats three of the cookies before they are split, there will be no leftovers when the cookies are split evenly between Laurel and Jean.
13. Is  $n/18$  an integer?  
 (1)  $5n/18$  is an integer. (2)  $3n/18$  is an integer.
14. If  $a$  and  $b$  are both single-digit positive integers, is  $a + b$  a multiple of 3?  
 (1) The two-digit number " $ab$ " (where  $a$  is in the tens place and  $b$  is in the ones place) is a multiple of 3.  
 (2)  $a - 2b$  is a multiple of 3.
15. The ratio of cupcakes to children at a particular birthday party is 104 to 7. Each child at the birthday party eats exactly  $x$  cupcakes (where  $x$  is a positive integer) and the adults attending the birthday party do not eat anything. If the number of cupcakes that remain uneaten is less than the number of children at the birthday party, what must be true about the number of uneaten cupcakes?  
 I. It is a multiple of 2. II. It is a multiple of 3. III. It is a multiple of 7.  
 (A) I only (B) II only (C) III only (D) I and II only (E) I, II and III
16. When the positive integer  $x$  is divided by 11, the quotient is  $y$  and the remainder 3. When  $x$  is divided by 19, the remainder is also 3. What is the remainder when  $y$  is divided by 19?  
 0 1 2 3 4
17.  $x$  is a positive number. If  $9^x + 9^{x+1} + 9^{x+2} + 9^{x+3} + 9^{x+4} + 9^{x+5} = y$ , is  $y$  divisible by 5?  
 1) 5 is a factor of  $x$ . 2)  $x$  is an integer.
18. A group of  $n$  students can be divided into equal groups of 4 with 1 student left over or equal groups of 5 with 3 students left over. What is the sum of the two smallest possible values of  $n$ ?  
 33 46 49 53 86
19. When  $x$  is divided by 4, the quotient is  $y$  and the remainder is 1. When  $x$  is divided by 7, the quotient is  $z$  and the remainder is 6. Which of the following is the value of  $y$  in terms of  $z$ ?  
 $(4z/7) + 5$   $(7z + 5)/6$   $(6z + 7)/4$   $(7z + 5)/4$   $(4z + 6)/7$
20. If  $n$  is an integer and  $n^4$  is divisible by 32, which of the following could be the remainder when  $n$  is divided by 32?  
 (A) 2 (B) 4 (C) 5 (D) 6 (E) 10
21.  $x_1$  and  $x_2$  are each positive integers. When  $x_1$  is divided by 3, the remainder is 1, and when  $x_2$  is divided by 12, the remainder is 4. If  $y = 2x_1 + x_2$ , then what must be true about  $y$ ?  
 I.  $y$  is even II.  $y$  is odd III.  $y$  is divisible by 3  
 (A) I only (B) II only (C) III only  
 (D) I and III only (E) II and III only
22. Is  $x$  the square of an integer?  
 (1)  $x = 12k + 6$ , where  $k$  is a positive integer  
 (2)  $x = 3q + 9$ , where  $q$  is a positive integer
23. If  $r - s = 3p$ , is  $p$  an integer? (1)  $r$  is divisible by 735 (2)  $r + s$  is divisible by 3
24. If  $n$  is a positive integer, is  $n^2 - 1$  divisible by 24?  
 (1)  $n$  is a prime number (2)  $n$  is greater than 191
25. The sum of all the digits of the positive integer  $q$  is equal to the three-digit number  $x13$ . If  $q = 10^n - 49$ , what is the value of  $n$ ?  
 (A) 24 (B) 25 (C) 26 (D) 27 (E) 28
26. Given that  $n$  is an integer; is  $n - 1$  divisible by 3?  
 (1)  $n^2 + n$  is not divisible by 3 (2)  $3n + 5 \geq k + 8$ , where  $k$  is a positive multiple of 3
27. Given that both  $x$  and  $y$  are positive integers, and that  $y = 3^{(x-1)} - x$ , is  $y$  divisible by 6?  
 (1)  $x$  is a multiple of 3 (2)  $x$  is a multiple of 4

28. If  $m$  and  $n$  are nonzero integers, is  $m/n$  an integer?  
 (1)  $2m$  is divisible by  $n$  (2)  $m$  is divisible by  $2n$
29. If positive integer  $n$  is divisible by both 4 and 21, then  $n$  must be divisible by which of the following?  
 8                      12                      18                      24                      48
30. Susie can buy apples from two stores: a supermarket that sells apples only in bundles of 4, and a convenience store that sells single, unbundled apples. If Susie wants to ensure that the total number of apples she buys is a multiple of 5, what is the minimum number of apples she must buy from the convenience store?  
 0                      1                      2                      3                      4
31. Each of the following numbers has a remainder of 2 when divided by 11 except:  
 2                      13                      24                      57                      185
32. When positive integer  $n$  is divided by 3, the remainder is 2; and when positive integer  $t$  is divided by 5, the remainder is 3. What is the remainder when the product  $nt$  is divided by 15?  
 1).  $n-2$  is divisible by 5                      2).  $t$  is divisible by 3
33. If  $n$  is a positive integer and  $r$  is the remainder when  $(n-1)(n+1)$  is divided by 24, what is the value of  $r$ ?  
 1).  $n$  is not divisible by 2                      2).  $n$  is not divisible by 3
34. If  $n$  is a positive integer and  $r$  is the remainder when  $n^2 - 1$  is divided by 8, what is the value of  $r$ ?  
 1).  $n$  is odd                      2).  $n$  is not divisible by 8
35. If  $n$  is a positive integer and  $r$  is the remainder when  $4+7n$  is divided by 3, what is the value of  $r$ ?  
 1).  $n+1$  is divisible by 3                      2).  $n > 20$
36. If  $r$  is the remainder when integer  $n$  is divided by 7, what is the value of  $r$ ?  
 1). When  $n$  is divided by 21, the remainder is an odd number  
 2). When  $n$  is divided by 28, the remainder is 3
37. What is the remainder when the positive integer  $x$  is divided by 6?  
 1). When  $x$  is divided by 2, the remainder is 1; and when  $x$  is divided by 3, the remainder is 0  
 2). When  $x$  is divided by 12, the remainder is 3.
38. When the positive integer  $x$  is divided by 11, the quotient is  $y$  and the remainder 3. When  $x$  is divided by 19, the remainder is also 3. What is the remainder when  $y$  is divided by 19?  
 0                      1                      2                      3                      4
39. When  $x$  is divided by 4, the quotient is  $y$  and the remainder is 1. When  $x$  is divided by 7, the quotient is  $z$  and the remainder is 6. What is the value of  $y$  in terms of  $z$ ?

### Factors, Divisors, Multiples, LCM, HCF

1. If  $n$  is a non-negative integer such that  $12^n$  is a divisor of 3,176,793, what is the value of  $n^{12} - 12^n$ ?  
 - 11                      - 1                      0                      1                      11
2. If the square root of  $p^2$  is an integer, which of the following must be true?  
 I.  $p^2$  has an odd number of factors  
 II.  $p^2$  can be expressed as the product of an even number of prime factors  
 III.  $p$  has an even number of factors  
 I                      II                      III                      I and II                      II and III
3. The greatest common factor of 16 and the positive integer  $n$  is 4, and the greatest common factor of  $n$  and 45 is 3. Which of the following could be the value of  $n$ ?  
 6                      8                      9                      12                      15
4. If  $x$  is a positive integer, is  $x-1$  a factor of 104?  
 (1)  $x$  is divisible by 3.                      (2) 27 is divisible by  $x$ .
5. How many factors does  $36^2$  have?



6. In a certain game, a large bag is filled with blue, green, purple and red chips worth 1, 5,  $x$  and 11 points each, respectively. The purple chips are worth more than the green chips, but less than the red chips. A certain number of chips are then selected from the bag. If the product of the point values of the selected chips is 88,000, how many purple chips were selected?  
1                      2                      3                      4                      5
7. For any integer  $k > 1$ , the term "length of an integer" refers to the number of positive prime factors, not necessarily distinct, whose product is equal to  $k$ . For example, if  $k = 24$ , the length of  $k$  is equal to 4, since  $24 = 2 \times 2 \times 2 \times 3$ . If  $x$  and  $y$  are positive integers such that  $x > 1$ ,  $y > 1$ , and  $x + 3y < 1000$ , what is the maximum possible sum of the length of  $x$  and the length of  $y$ ?  
5                      6                      15                      16                      18
9. If  $a$  and  $b$  are positive integers divisible by 6, is 6 the greatest common divisor of  $a$  and  $b$ ?  
(1)  $a = 2b + 6$                       (2)  $a = 3b$
10.  $a$ ,  $b$ , and  $c$  are positive integers. If  $a$ ,  $b$ , and  $c$  are assembled into the six-digit number  $abcabc$ , which one of the following must be a factor of  $abcabc$ ?  
(A) 16                      (B) 13                      (C) 5                      (D) 3                      (E) none of the above
11. If  $x$  and  $y$  are positive integers, which of the following CANNOT be the greatest common divisor of  $35x$  and  $20y$ ?  
5                       $5(x - y)$                        $20x$                        $20y$                        $35x$
12. If  $P$ ,  $Q$ ,  $R$ , and  $S$  are positive integers, and  $P/Q = R/S$ , is  $R$  divisible by 5 ?  
(1)  $P$  is divisible by 140                      (2)  $Q = 7^x$ , where  $x$  is a positive integer
13. For any four digit number,  $abcd$ ,  $*abcd* = (3^a)(5^b)(7^c)(11^d)$ . What is the value of  $(n - m)$  if  $m$  and  $n$  are four-digit numbers for which  $*m* = (3^7)(5^9)(7^4)(11^6)$  and  $*n* = (25)(*m*)$ ?  
2000                      200                      25                      20                      2
14.  $w$ ,  $x$ ,  $y$ , and  $z$  are integers. If  $w > x > y > z > 0$ , is  $y$  a common divisor of  $w$  and  $x$ ?  
(1)  $\frac{w}{x} = z^{-1} + x^{-1}$                       (2)  $w^2 - wy - 2w = 0$
15. A restaurant pays a seafood distributor  $d$  dollars for 6 pounds of Maine lobster. Each pound can make  $v$  vats of lobster bisque, and each vat makes  $b$  bowls of lobster bisque. If the cost of the lobster per bowl is an integer, and if  $v$  and  $b$  are different prime integers, then which of the following is the smallest possible value of  $d$ ?  
(A) 15                      (B) 24                      (C) 36                      (D) 54                      (E) 90
16.  $a$ ,  $b$ ,  $c$ , and  $d$  are positive integers. If  $(a + b)(c - d) = r$ , where  $r$  is an integer, is  $\sqrt{c + d}$  an integer?  
(1)  $(a + b)(c + d) = r^2$   
(2)  $(a + b) = x^4 y^6 z^2$ , where  $x$ ,  $y$ , and  $z$  are distinct prime numbers.
17. The function  $f(n)$  = the number of factors of  $n$ . If  $f(pq) = 4$ , what is the value of the integer  $p$ ?  
(1)  $p + q$  is an odd integer                      (2)  $q < p$
18. If  $x$ ,  $y$ , and  $z$  are positive integers such that  $x < y < z$ , is  $x$  a factor of the odd integer  $z$ ?  
(1)  $x$  and  $y$  are prime numbers, whose sum is a factor of 57                      (2)  $z$  is a factor of 57
19. What is the positive integer  $n$ ?  
(1) The sum of all of the positive factors of  $n$  that are less than  $n$  is equal to  $n$   
(2)  $n < 30$
20. If  $p^3$  is divisible by 80, then the positive integer  $p$  must have at least how many distinct factors?  
(A) 2    (B) 3    (C) 6    (D) 8    (E) 10
21. Does the integer  $p$  have an odd number of distinct factors?  
(1)  $p = q^2$ , where  $q$  is a nonzero integer.    (2)  $p = 2n + 1$ , where  $n$  is a nonzero integer.

22. What is the positive integer  $n$ ?  
 (1) For every positive integer  $m$ , the product  $m(m+1)(m+2) \dots (m+n)$  is divisible by 16  
 (2)  $n^2 - 9n + 20 = 0$
23. What is the greatest common factor of positive integers  $a$  and  $b$ ?  
 (1)  $a = b + 4$  (2)  $b/4$  is an integer
24. Which of the following is the lowest positive integer that is divisible by the first 7 positive integer multiples of 5?  
 140                      210                      1400                      2100                      3500
25. What is the value of the integer  $n$ ?  
 (1)  $n! = n \times (n-1)!$  (2)  $n^3 + 3n^2 + 2n$  is divisible by 3
26.  $p^a q^b r^c s^d = x$ , where  $x$  is a perfect square. If  $p, q, r$ , and  $s$  are prime integers, are they distinct?  
 (1) 18 is a factor of  $ab$  and  $cd$  (2) 4 is not a factor of  $ab$  and  $cd$
27.  $K$  and  $L$  are each four-digit positive integers with thousands, hundreds, tens, and units digits defined as  $a, b, c$ , and  $d$ , respectively, for the number  $K$ , and  $p, q, r$ , and  $s$ , respectively, for the number  $L$ . For numbers  $K$  and  $L$ , the function  $W$  is defined as  $5^a 2^b 7^c 3^d \div 5^p 2^q 7^r 3^s$ . The function  $Z$  is defined as  $(K - L) \div 10$ . If  $W = 16$ , what is the value of  $Z$ ?  
 (A) 16 (B) 20 (C) 25 (D) 40 (E) It cannot be determined from the information given.
28. How many numbers that are not divisible by 6 divide evenly into 264,600?  
 (A) 9 (B) 36 (C) 51 (D) 63 (E) 72
29. If  $n^2 / n$  yields an integer greater than 0, is  $n$  divisible by 30?  
 (1)  $n^2$  is divisible by 20 (2)  $n^3$  is divisible by 12
30. If  $a$  and  $b$  are **consecutive positive integers**, and  $ab = 30x$  is  $x$  a non-integer?  
 (1)  $a^2$  is divisible by 21 (2) 35 is a factor of  $b^2$
31.  $\sqrt{ABC} = 504$ . Is  $B$  divisible by 2? (1)  $C = 168$  (2)  $A$  is a perfect square
32. If the prime factorization of the integer  $q$  can be expressed as  $a^{2x} \cdot b^x \cdot c^{3x-1}$ , where  $a, b, c$ , and  $x$  are distinct positive integers, which of the following could be the total number of factors of  $q$ ?  
 (A)  $3j + 4$ , where  $j$  is a positive integer  
 (B)  $5k + 5$ , where  $k$  is a positive integer  
 (C)  $6l + 2$ , where  $l$  is a positive integer  
 (D)  $9m + 7$ , where  $m$  is a positive integer  
 (E)  $10n + 1$ , where  $n$  is a positive integer
33. Which of the following is the lowest positive integer that is divisible by 8, 9, 10, 11, and 12?  
 7,920                      5,940                      3,960                      2,970                      890
34. If  $x$  is a positive integer, is  $x$  prime?  
 (1)  $x$  has the same number of factors as  $y^2$ , where  $y$  is a positive integer greater than 2.  
 (2)  $x$  has the same number of factors as  $z$ , where  $z$  is a positive integer greater than 2.
35.  $h(n)$  is the product of the even numbers from 2 to  $n$ , inclusive, and  $p$  is the least prime factor of  $h(100)+1$ . What is the range of  $p$ ?  
 < 40                      < 30                      > 40                      < 10                      Indeterminate
36. If  $d$  is positive integer,  $f$  is the product of the first 30 positive integers, what is the value of  $d$ ?  
 1).  $10^d$  is a factor of  $f$  2).  $d > 6$
37. Does the integer  $k$  have a factor  $p$  such that  $1 < p < k$ ?  
 1).  $k > 4!$  2).  $13! + 2 \leq k \leq 13! + 13$
38. If  $x$  and  $y$  are integers greater than 1, is  $x$  a multiple of  $y$ ?  
 1).  $3y^2 + 7y = x$  2).  $x^2 - x$  is a multiple of  $y$

39. The function  $f$  is defined for all positive integers  $n$  by the following rule.  $f(n)$  is the number of positive integers each of which is less than  $n$  and has no positive factor in common with  $n$  other than 1. If  $p$  is any prime number then  $f(p) =$   
 $p-1$      $p-2$      $(p+1)/2$      $(p-1)/2$     2
40. In the fraction  $x/y$ , where  $x$  and  $y$  are positive integers, what is the value of  $y$ ?  
 1). The least common denominator of  $x/y$  and  $1/3$  is 6    2).  $x=1$
41. For any positive integer  $n$ , the length of  $n$  is defined as number of prime factors whose product is  $n$ . For example, the length of 75 is 3, since  $75=3*5*5$ . How many two-digit positive integers have length 6?  
 0    1    2    3    4
42. If  $n$  and  $t$  are positive integers, what is the greatest prime factor of  $nt$ ?  
 1). The greatest common factor of  $n$  and  $t$  is 5  
 2). The least common multiple of  $n$  and  $t$  is 105
43. If  $n$  is a positive integer less than 200 and  $14n/60$  is an integer, then  $n$  has how many different positive prime factors?  
 A. two    B. three    C. five    D. six    E. eight.
44. The positive integers  $x$ ,  $y$  and  $z$  are such that  $x$  is a factor of  $y$  and  $y$  is a factor of  $z$ . Is  $z$  even?  
 1).  $xz$  is even    2).  $y$  is even
45. If  $k$  is a positive integer, then  $20k$  is divisible by how many different positive integers?  
 1).  $k$  is prime.    2).  $k=7$
46.  $x$  and  $y$  are positive integers such that  $x=8y+12$ , what is the greatest common divisor of  $x$  and  $y$ ?  
 1).  $X=12u$ , where  $u$  is an integer.    2).  $Y=12z$ , where  $z$  is an integer.
47. What is the greatest prime factor of  $4^{17} - 2^{28}$ ?
48. How many different prime numbers are factors of the positive integer  $n$ ?  
 1). four different prime numbers are factors of  $2n$ .  
 2). four different prime numbers are factors of  $n^2$ .
49. What is the greatest common factor of positive integers  $a$  and  $b$ ?  
 (1)  $a = b + 4$     (2)  $b/4$  is an integer
50. Which of the following is the lowest positive integer that is divisible by the first 7 positive integer multiples of 5?  
 140    210    1400    2100    3500

### Consecutive Integers

1.  $x$  is the sum of  $y$  consecutive integers.  $w$  is the sum of  $z$  consecutive integers. If  $y = 2z$ , and  $y$  and  $z$  are both positive integers, then each of the following could be true EXCEPT  
 $x = w$      $x > w$      $x/y$  is an integer     $w/z$  is an integer  
 $x/z$  is an integer
2. For positive integer  $k$ , is the expression  $(k+2)(k^2 + 4k + 3)$  divisible by 4?  
 (1)  $k$  is divisible by 8.    (2)  $(k+1)/3$  is an odd integer.
3. If  $x$  is an integer, then  $x(x-1)(x-k)$  must be evenly divisible by three when  $k$  is any of the following values EXCEPT  
 -4    -2    -1    2    5
4. The sum of  $n$  consecutive positive integers is 45. What is the value of  $n$ ?  
 (1)  $n$  is even    (2)  $n < 9$
5. Is positive integer  $n-1$  a multiple of 3?  
 (1)  $n^3 - n$  is a multiple of 3    (2)  $n^3 + 2n^2 + n$  is a multiple of 3

6.  $a, b, c,$  and  $d$  are consecutive integers and  $a < b < c < d$ . If the product of  $b, c,$  and  $d$  is twice that of  $a, b,$  and  $c$ , then  $bc =$   
 2                      6                      12                      20                      30
8. How many integers are there between 51 and 107, inclusive?  
 51                      55                      56                      57                      58
9. If  $x, y,$  and  $z$  are 3 positive consecutive integers such that  $x < y < z$ , what is the remainder when the product of  $x, y,$  and  $z$  is divided by 8?  
 (1)  $(xz)^2$  is even                      (2)  $5y^3$  is odd
10. If  $x^3 - x = n$  and  $x$  is a positive integer greater than 1, is  $n$  divisible by 8?  
 (1) When  $3x$  is divided by 2, there is a remainder.  
 (2)  $x = 4y + 1$ , where  $y$  is an integer.
11.  $a$  is the sum of  $x$  consecutive positive integers.  $b$  is the sum of  $y$  consecutive positive integers. For which of the following values of  $x$  and  $y$  is it impossible that  $a = b$ ?  
 (A)  $x = 2; y = 6$                       (B)  $x = 3; y = 6$                       (C)  $x = 7; y = 9$   
 (D)  $x = 10; y = 4$                       (E)  $x = 10; y = 7$
12. Is  $x$  divisible by 30?  
 (1)  $x = k(m^3 - m)$ , where  $m$  and  $k$  are both integers  $> 9$   
 (2)  $x = n^5 - n$ , where  $n$  is an integer  $> 9$
13. If  $x, y,$  and  $z$  are positive integers, where  $x > y$  and  $z = \sqrt{x}$ , are  $x$  and  $y$  consecutive perfect squares?  
 (1)  $x + y = 8z + 1$                       (2)  $x - y = 2z - 1$

### Digits

1. Given that  $a, b, c,$  and  $d$  are different nonzero digits and that  $10d + 11c < 100 - a$ , which of the following could *not* be a solution to the addition problem below?  

$$\begin{array}{r} abdc \\ + dbca \\ \hline \end{array}$$
  
 (A) 3689                      (B) 6887                      (C) 8581                      (D) 9459                      (E) 16091
2. 
$$\begin{array}{r} 8k8 \\ + k88 \\ \hline 1,6p6 \end{array}$$
  
 If  $k$  and  $p$  represent non-zero digits within the integers above, what is  $p$ ?  
 6                      7                      8                      9                      17
3. If  $x$  represents the sum of all the positive three-digit numbers that can be constructed using each of the distinct nonzero digits  $a, b,$  and  $c$  exactly once, what is the largest integer by which  $x$  must be divisible?  
 (A) 3                      (B) 6                      (C) 11                      (D) 22                      (E) 222
4. If the sum of the digits of the positive two-digit number  $x$  is 4, what is the value of  $x$ ?  
 (1)  $x$  is odd.                      (2) Twice the value of  $x$  is less than 44.
5. 
$$\begin{array}{r} 22 \\ a3 \\ + 4b \\ \hline 90 \end{array}$$
  
 If  $a$  and  $b$  represent positive single digits in the correctly worked computation above, what is the value of the two digit integer  $ba$ ?  
 10                      15                      25                      51                      52

6. 
$$\begin{array}{r} a b c \\ + d e f \\ \hline x y z \end{array}$$

If, in the addition problem above,  $a, b, c, d, e, f, x, y$ , and  $z$  each represent different positive single digits, what is the value of  $z$ ?

(1)  $3a = f = 6y$       (2)  $f - c = 3$

7. For any four digit number,  $abcd$ ,  $*abcd* = (3^a)(5^b)(7^c)(11^d)$ . What is the value of  $(n - m)$  if  $m$  and  $n$  are four-digit numbers for which  $*m* = (3^7)(5^5)(7^4)(11^6)$  and  $*n* = (25)(*m*)$ ?
- 2000    200    25    20    2

8. What is the three-digit number  $abc$ , given that  $a, b$ , and  $c$  are the positive single digits that make up the number?

(1)  $a = 1.5b$  and  $b = 1.5c$

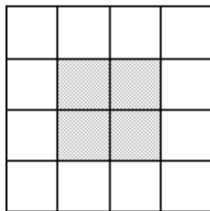
(2)  $a = 1.5x + b$  and  $b = x + c$ , where  $x$  represents a positive single digit

9. What is the value of the three-digit number  $SSS$  if  $SSS$  is the sum of the three-digit numbers  $ABC$  and  $XYZ$ , where each letter represents a distinct digit from 0 to 9, inclusive?

1)  $S = 1.75X$

2)  $S^2 = 49zx/8$

10. If the  $4 \times 4$  grid pictured at right is filled with the consecutive integers from 37 to 52, inclusive, so that every row, column and major diagonal sums to the same value, which of the following is a possible value of the sum of the four center cells of the grid (indicated by the shaded area)?



(A) 124

(B) 153

(C) 178

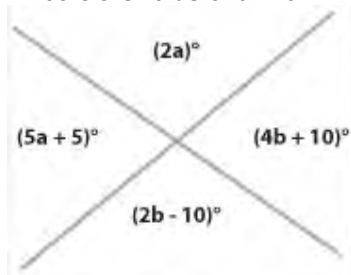
(D) 192

(E) 214

# GMAT Quant Topic 5: Geometry

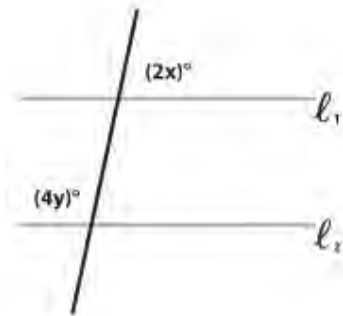
## Part 1: Lines and Angles

1. What is the value of  $a + b$ ?



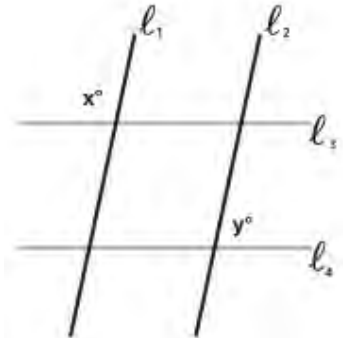
30      50      55      65      90

2. If  $l_1$  is parallel to  $l_2$ , what is  $x + 2y$ ?



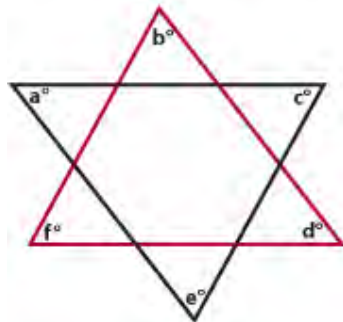
90      120      180      270      360

3. What is the value of  $x$ ?



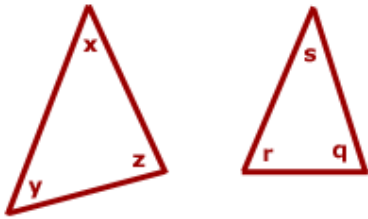
(1)  $l_1$  is parallel to  $l_2$       (2)  $y = 70$

4. What is the value of  $a + b + c + d + e + f$ ?



180      270      300      360      720

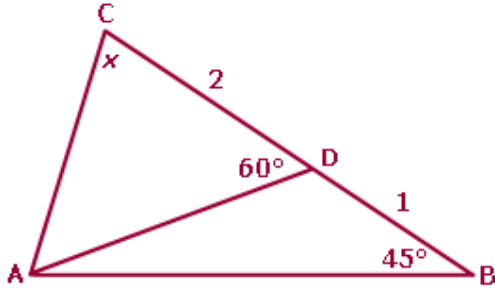
5. If  $x - q = s - y$ , what is the value of  $z$ ?



**Figures are not drawn to scale.**

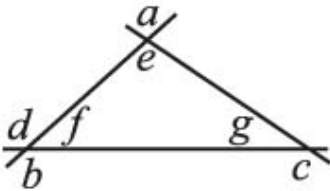
1)  $xq + sy + sx + yq = zr$       2)  $zq - ry = rx - zs$

6. In the figure, point  $D$  divides side  $BC$  of triangle  $ABC$  into segments  $BD$  and  $DC$  of lengths 1 and 2 units respectively. Given that  $\angle ADC = 60^\circ$  and  $\angle ABD = 45^\circ$ , what is the measure of angle  $x$  in degrees? (Note: Figure is not drawn to scale.)



- (A) 55      (B) 60      (C) 70      (D) 75      (E) 90

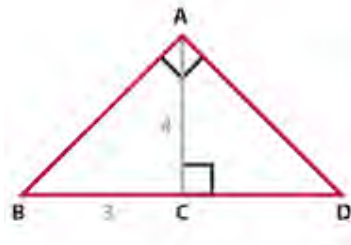
7. What is the degree measure of angle  $a$ ?



- (1)  $b + c = 287$  degrees      (2)  $d + e = 269$  degrees

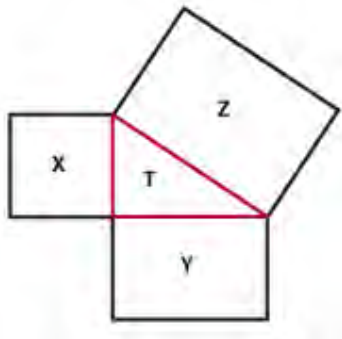
## Topic 2: Triangles

1. In triangle  $ABC$ , if  $BC = 3$  and  $AC = 4$ , then what is the length of segment  $CD$ ?



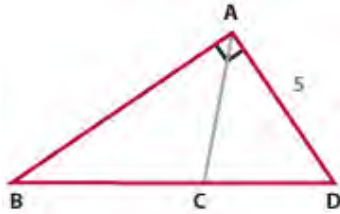
- 3       $15/4$       5       $16/3$        $20/3$

2. The figure is comprised of three squares and a triangle. If the areas marked  $X$ ,  $Y$  and  $Z$  are 25, 144, and 169, respectively, what is the area of the triangle marked  $T$ ?



25      30      50      60      97

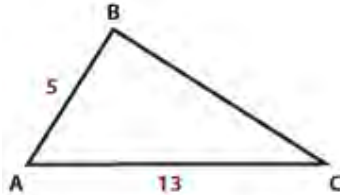
3. If angle  $BAD$  is a right angle, what is the length of side  $BD$ ?



(1)  $AC$  is perpendicular to  $BD$

(2)  $BC = CD$

4. What is the length of segment  $BC$ ?



(1) Angle  $ABC$  is 90 degrees.

(2) The area of the triangle is 30.

5. What is the perimeter of isosceles triangle  $ABC$ ?

(1) The length of side  $AB$  is 9

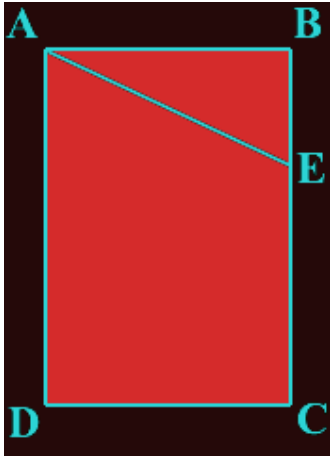
(2) The length of side  $BC$  is 4

6. The figure is made up of a series of inscribed equilateral triangles. If the pattern continues until the length of a side of the largest triangle (i.e. the entire figure) is exactly 128 times that of the smallest triangle, what fraction of the total figure will be shaded?



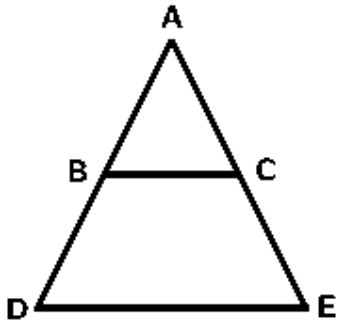


7. Given that ABCD is a rectangle, is the area of triangle ABE  $> 25$ ?  
(Note: Figure above is not drawn to scale).



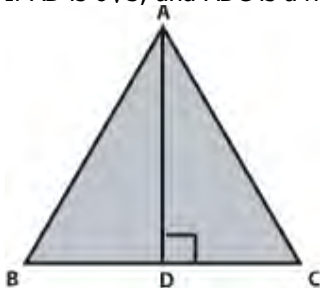
(1)  $AB = 6$       (2)  $AE = 10$

8. In the figure,  $AC = 3$ ,  $CE = x$ , and  $BC$  is parallel to  $DE$ . If the area of



triangle ABC is  $\frac{1}{12}$  of the area of triangle ADE, then  $x = ?$

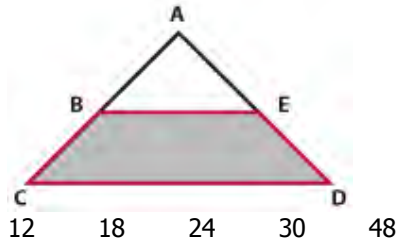
9. Triangle A has one side of length  $x$ . If  $\sqrt{(x^8)} = 81$ , what is the perimeter of Triangle A?  
1) Triangle A has sides whose lengths are consecutive integers  
2) Triangle A is NOT a right triangle
10. If  $AD$  is  $6\sqrt{3}$ , and  $ADC$  is a right angle, what is the area of triangular region ABC?



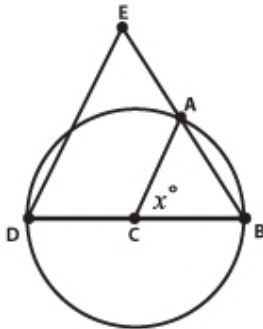
(1)  $\angle ABD = 60^\circ$

(2)  $AC = 12$

11. If  $BE \parallel CD$ , and  $BC = AB = 3$ ,  $AE = 4$  and  $CD = 10$ , what is the area of trapezoid  $BEDC$ ?

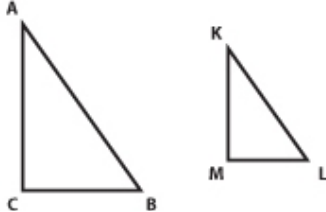


12. If the length of side  $AB$  is 17, is triangle  $ABC$  a right triangle?  
 (1) The length of side  $BC$  is 144. (2) The length of side  $AC$  is 145.
13. In the figure, if point  $C$  is the center of the circle and  $DB = 7$ , what is the length of  $DE$ ?

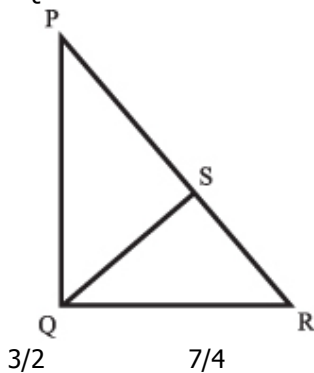


- (1)  $x = 60^\circ$  (2)  $DE \parallel CA$

14. The area of the right triangle  $ABC$  is 4 times greater than the area of the right triangle  $KLM$ . If the hypotenuse  $KL$  is 10 inches, what is the length of the hypotenuse  $AB$ ?

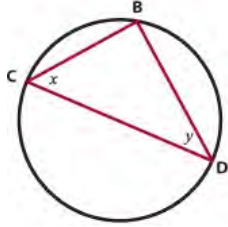


- (1) Angles  $ABC$  and  $KLM$  are each equal to 55 degrees.  
 (2)  $LM$  is 6 inches.
15. In the diagram, triangle  $PQR$  has a right angle at  $Q$  and a perimeter of 60. Line segment  $QS$  is perpendicular to  $PR$  and has a length of 12.  $PQ > QR$ . What is the ratio of the area of triangle  $PQS$  to the area of triangle  $RQS$ ?



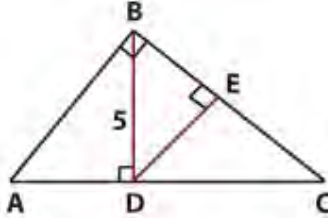
- $\frac{3}{2}$   $\frac{7}{4}$   $\frac{15}{8}$   $\frac{16}{9}$  2

16. If  $CD$  is the diameter of the circle, does  $x$  equal 30?



- (1) The length of  $CD$  is twice the length of  $BD$ . (2)  $y = 60$

17. In the diagram, what is the length of  $AB$ ?

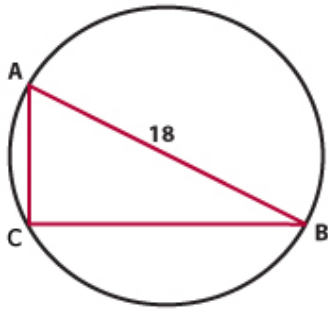


- (1)  $BE = 3$  (2)  $DE = 4$

18. Which of the following is a possible length for side  $AB$  of triangle  $ABC$  if  $AC = 6$  and  $BC = 9$ ?

I. 3 II.  $9\sqrt{3}$  III. 13.5  
I only II only III only II and III I, II and III

19. For the triangle shown, where  $A$ ,  $B$  and  $C$  are all points on a circle, and line segment  $AB$  has length 18, what is the area of triangle  $ABC$ ?



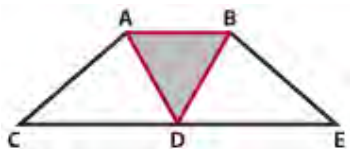
- (1) Angle  $ABC$  measures  $30^\circ$ . (2) The circumference of the circle is  $18\pi$ .

20. The perimeter of a certain isosceles right triangle is  $16 + 16\sqrt{2}$ , what is the length of the hypotenuse of the triangle?

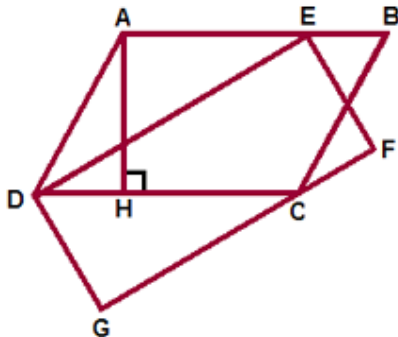
### Topic 3: Quadrilaterals

- Is quadrilateral  $ABCD$  a rectangle?  
(1) Line segments  $AC$  and  $BD$  bisect one another.  
(2) Angle  $ABC$  is a right angle.
- Is quadrilateral  $ABCD$  a rhombus?  
(1) Line segments  $AC$  and  $BD$  are perpendicular bisectors of each other.  
(2)  $AB = BC = CD = AD$
- Is quadrilateral  $ABCD$  a square?  
(1)  $ABCD$  is a rectangle. (2)  $AB = BC$
- Rectangle  $ABCD$  is inscribed in circle  $P$ . What is the area of circle  $P$ ?  
(1) The area of rectangle  $ABCD$  is 100. (2) Rectangle  $ABCD$  is a square.

5. If triangle  $ABD$  is an equilateral triangle and  $AB = 6$  and  $CE = 18$ , what fraction of the trapezoid  $BACE$  is shaded?

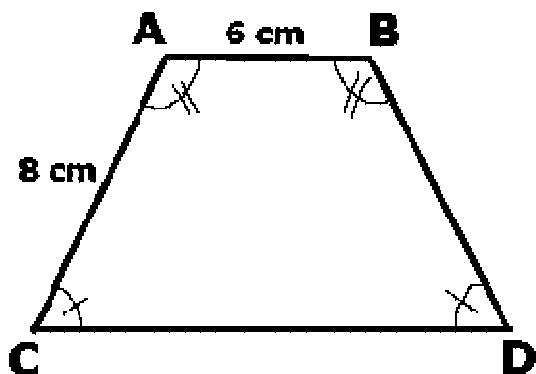


6. In the picture, quadrilateral  $ABCD$  is a parallelogram and quadrilateral  $DEFG$  is a rectangle. What is the area of parallelogram  $ABCD$  (figure not drawn to scale)?



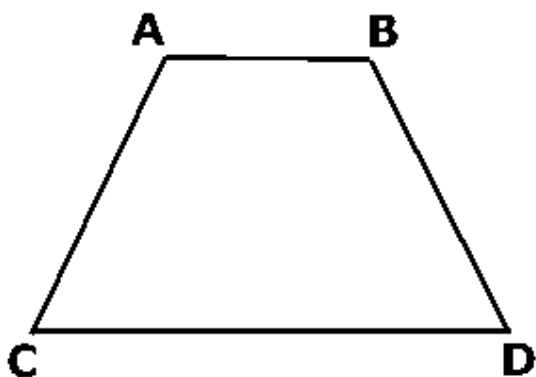
- (1) The area of rectangle  $DEFG$  is  $8\sqrt{5}$ . (2) Line  $AH$ , the altitude of parallelogram  $ABCD$ , is 5.

7. What is the area of the trapezoid shown?



- (1) Angle  $A = 120$  degrees (2) The perimeter of trapezoid  $ABCD = 36$ .

8. The height of isosceles trapezoid  $ABDC$  is 12 units. The length of diagonal  $AD$  is 15 units. What is the area of trapezoid  $ABDC$ ?



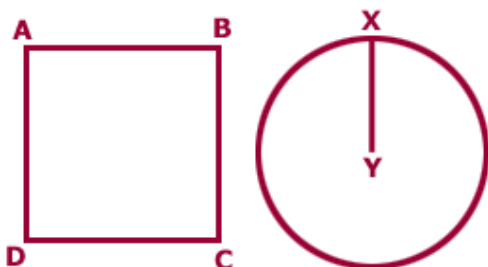
- (A) 72 (B) 90 (C) 96 (D) 108 (E) 180

9. The combined area of the two black squares is equal to 1000 square units. A side of the larger black square is 8 units longer than a side of the smaller black square. What is the combined area of the two white rectangles in square units?



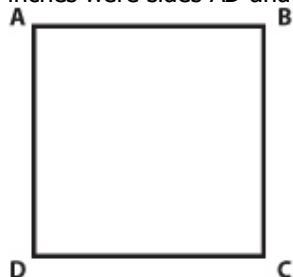
(A) 928 (B) 936 (C) 948 (D) 968 (E) 972

10. Jeff is painting two murals on the front of an old apartment building that he is renovating. One mural will be in the shape of a square  $ABCD$  and the other will be in the shape of a circle  $XY$ . Assuming that the thickness of the coats of paint is negligible; will each mural require the same amount of paint? **Note:** Figures are not drawn to scale.



(1)  $AB = BC = CD = DA$ , and  $AB = XY\sqrt{2}$       (2)  $AC = BD$  and  $AC = XY\sqrt{2}$

11. In the quadrilateral  $PQRS$ , side  $PS$  is parallel to side  $QR$ . Is  $PQRS$  a parallelogram?  
 (1)  $PS = QR$       (2)  $PQ = RS$
12.  $E, F, G$ , and  $H$  are the vertices of a polygon. Is polygon  $EFGH$  a square?  
 (1)  $EFGH$  is a parallelogram.  
 (2) The diagonals of  $EFGH$  are perpendicular bisectors of one another.
13. What is the area of the quadrilateral with vertices  $A, B, C$ , and  $D$ ?  
 (1) The perimeter of  $ABCD$  is equal to 16.  
 (2) Quadrilateral  $ABCD$  is a square.
14. The perimeter of a rectangular yard is completely surrounded by a fence that measures 40 meters. What is the length of the yard if the area of the yard is 64 meters squared?  
 8      10      12      14      16
15. Square  $ABCD$  has an area of 9 square inches. Sides  $AD$  and  $BC$  are lengthened to  $x$  inches each. By how many inches were sides  $AD$  and  $BC$  lengthened?



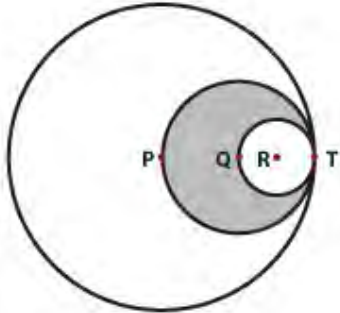
(1) The diagonal of the resulting rectangle measures 5 inches.  
 (2) The resulting rectangle can be cut into three rectangles of equal size.

16. In the rhombus  $ABCD$ , the length of diagonal  $BD$  is 6 and the length of diagonal  $AC$  is 8. What is the perimeter of  $ABCD$ ?  
 10      14      20      24      28

17. Is the measure of one of the interior angles of quadrilateral ABCD equal to 60 degrees?  
 1). two of the interior angles of ABCD are right angles  
 2). the degree measure of angle ABC is twice the degree measure of angle BCD

#### Topic 4: Circles

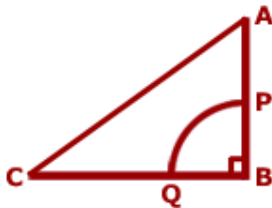
1. If  $P$ ,  $Q$  and  $R$  are the centers of circles  $P$ ,  $Q$ , and  $R$  and the points  $P$ ,  $Q$ ,  $R$  and  $T$  all lie on the same line, what portion of circle  $P$  is shaded?



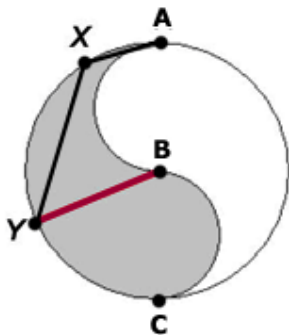
2. If  $\frac{1}{a^2} + a^2$  represents the diameter of circle  $O$  and  $\frac{1}{a} + a = 3$ , which of the following best approximates the circumference of circle  $O$ ?  
 28      22      20      16      12
3. A car is being driven on a road. Assuming that the car's wheels turn without slipping, how many full  $360^\circ$  rotations does each tire on the car make in 10 minutes?  
 (1) The car is traveling at 50 miles per hour.  
 (2) Each tire has a radius of 20 inches.
4. Two circular road signs are to be painted yellow. If the radius of the larger sign is twice that of the smaller sign, how many times more paint is needed to paint the larger sign (assuming that a given amount of paint covers the same area on both signs)?  
 2      3      4       $3/2$
5. The figure represents five concentric quarter-circles. The length of the radius of the largest quarter-circle is  $x$ . The length of the radius of each successively smaller quarter-circle is one less than that of the next larger quarter-circle. What is the combined area of the shaded regions (black), in terms of  $x$ ?



6. In the diagram (not drawn to scale), Sector PQ is a quarter-circle. The distance from A to P is half the distance from P to B. The distance from C to Q is  $\frac{2}{7}$  of the distance from Q to B. If the length of AC is 100, what is the length of the radius of the circle with center B?

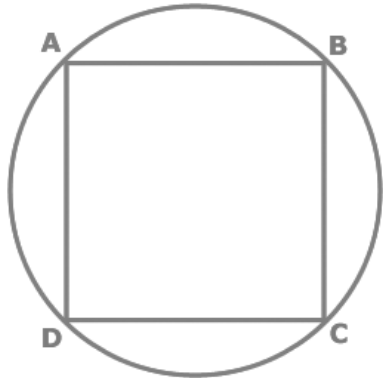


7. A circular gear with a diameter of 24 centimeters is mounted directly on another circular gear with a diameter of 96 centimeters. Both gears turn on the same axle at their exact centers and each gear has a single notch, at the 12 o'clock position. At the same moment, the gears begin to turn at the same rate, with the larger gear moving clockwise and the smaller gear counterclockwise. How far, in centimeters, will the notch on the larger gear have traveled the second time the notches pass each other?
8. In the diagram, points  $A$ ,  $B$ , and  $C$  are on the diameter of the circle with center  $B$ . Additionally, all arcs pictured are semicircles. Suppose angle  $YXA = 105$  degrees. What is the ratio of the area of the shaded region above the line  $YB$  to the area of the shaded region below the line  $YB$ ? (Note: Diagram is not drawn to scale and angles drawn are not accurate.)

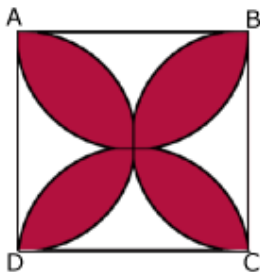


- (A)  $\frac{3}{4}$       (B)  $\frac{5}{6}$       (C) 1      (D)  $\frac{7}{5}$       (E)  $\frac{9}{7}$

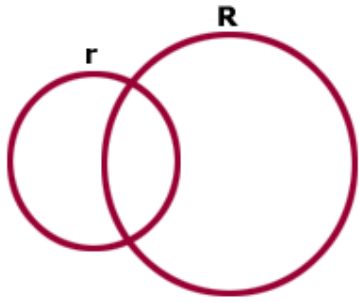
9. For a circle with center point  $P$ , cord  $XY$  is the perpendicular bisector of radius  $AP$  ( $A$  is a point on the edge of the circle). What is the length of cord  $XY$ ?  
 (1) The circumference of circle  $P$  is twice the area of circle  $P$ .      (2) The length of Arc  $XAY = 2\sqrt{3}$ .
10.  $ABCD$  is a square inscribed in a circle and arc  $ADC$  has a length of  $\sqrt{x}$ . If a dart is thrown and lands somewhere in the circle, what is the probability that it will not fall within the inscribed square? (Assume that the point in the circle where the dart lands is completely random.)



11. Figure  $ABCD$  is a square with sides of length  $x$ . Arcs  $AB$ ,  $AD$ ,  $BC$ , and  $DC$  are all semicircles. What is the area of the black region, in terms of  $x$ ?

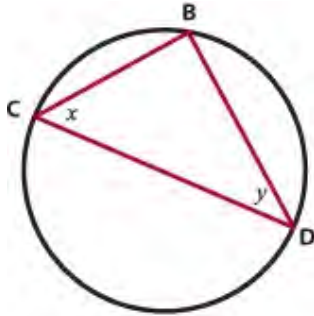


12. In the figure, a small circle with radius  $r$  intersects a larger circle with radius  $R$  (where  $R > r$ ). If  $k > 0$ , what is the difference in the areas of the non-overlapping parts of the two circles?



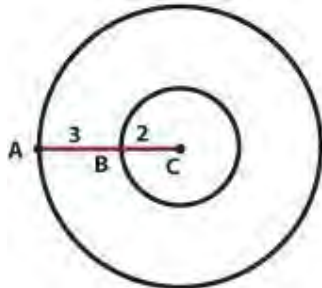
- (1)  $R = r + 3k$       (2)  $(kR) / (kr - 6) = -1$

13. If  $CD$  is the diameter of the circle, does  $x$  equal 30?



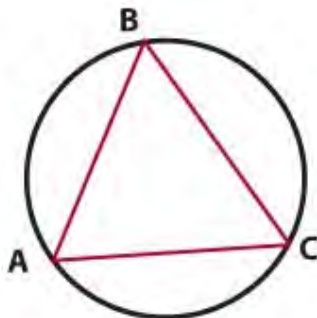
- (1) The length of  $CD$  is twice the length of  $BD$ .      (2)  $y = 60$

14. Two circles share a center at point  $C$ , as shown. Segment  $AC$  is broken up into two shorter segments,  $AB$  and  $BC$ , with dimensions shown. What is the ratio of the area of the large circle to the area of the small circle?



25/4    5/2    3/2    2/5    4/25

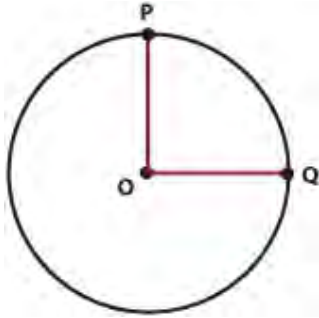
15. The length of minor arc  $AB$  is twice the length of minor arc  $BC$  and the length of minor arc  $AC$  is three times the length of minor arc  $AB$ . What is the measure of angle  $BCA$ ?



20                  40                  60                  80                  120

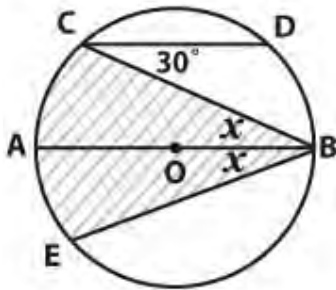


16. What is the radius of the circle shown?



- (1) The measure of arc  $PQ$  is  $4\pi$ . (2) The center of the circle is at point  $O$ .

17. A cylindrical tank has a base with a circumference of  $4\sqrt{\pi\sqrt{3}}$  meters and an equilateral triangle painted on the interior side of the base. A grain of sand is dropped into the tank, and has an equal probability of landing on any particular point on the base. If the probability of the grain of sand landing on the portion of the base outside the triangle is  $3/4$ , what is the length of a side of the triangle?
18. In the figure, circle  $O$  has center  $O$ , diameter  $AB$  and a radius of 5. Line  $CD$  is parallel to the diameter. What is the perimeter of the shaded region?



- $\frac{5}{3} + 5\sqrt{3}$        $\frac{5}{3} + 10\sqrt{3}$        $\frac{10}{3} + 5\sqrt{3}$   
 $\frac{10}{3} + 10\sqrt{3}$        $\frac{10}{3} + 20\sqrt{3}$

19. The figure shows the top side of a circular medallion made of a circular piece of colored glass surrounded by a metal frame, represented by the shaded region.



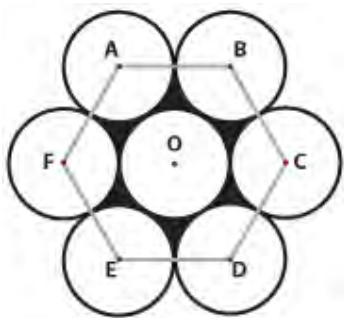
If the radius of the medallion is  $r$  centimeter and width of the metal frame is  $s$  centimeter, then, in terms of  $r$  and  $s$ , what is the area of the metal frame, in square centimeter?

20. A thin piece of wire 40 meters long is cut into two pieces. One piece is used to form a circle with radius  $r$ , and the other is used to form a square. No wire is left over. Which of the following represents the total area, in square meters, of the circular and the square regions in terms of  $r$ ?

### Topic 5: Polygons

1. A certain game board is in the shape of a non-convex polygon, with spokes that extend from each vertex to the center of the board. If each spoke is 8 inches long, and spokes are used nowhere else on the board, what is the sum of the interior angles of the polygon?
- (1) The sum of the exterior angles of the polygon is  $360^\circ$ .  
 (2) The sum of the exterior angles is equal to five times the total length of all of the spokes used.
2. The measures of the interior angles in a polygon are consecutive integers. The smallest angle measures 136 degrees. How many sides does this polygon have?
- A) **8**    B) **9**    C) **10**    D) **11**    E) **13**

3. If  $x$  represents the sum of the interior angles of a regular hexagon and  $y$  represents the sum of the interior angles of a regular pentagon, then the difference between  $x$  and  $y$  is equal to the sum of the interior angles of what geometric shape?  
 Triangle      Square      Rhombus      Trapezoid      Pentagon
4. If Polygon  $X$  has fewer than 9 sides, how many sides does Polygon  $X$  have?  
 (1) The sum of the interior angles of Polygon  $X$  is divisible by 16.  
 (2) The sum of the interior angles of Polygon  $X$  is divisible by 15.
5. Regular hexagon  $ABCDEF$  has a perimeter of 36.  $O$  is the center of the hexagon and of circle  $O$ . Circles  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$  have centers at  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$ , and  $F$ , respectively. If each circle is tangent to the two circles adjacent to it and to circle  $O$ , what is the area of the shaded region (inside the hexagon but outside the circles)?



$$108 - 18$$

$$108 - 27$$

$$54\sqrt{3} - 9$$

$$54\sqrt{3} - 27$$

$$54\sqrt{3} - 18$$

### Topic 6: General Solids (Cube, Box, Sphere)

1. Four spheres and three cubes are arranged in a line according to increasing volume, with no two solids of the same type adjacent to each other. The ratio of the volume of one solid to that of the next largest is constant. If the radius of the smallest sphere is  $\frac{1}{4}$  that of the largest sphere, what is the radius of the smallest sphere?  
 1) The volume of the smallest cube is 72 .  
 2) The volume of the second largest sphere is 576 .
2. At 7:57 am, Flight 501 is at an altitude of 6 miles above the ground and is on a direct approach (i.e., flying in a direct line to the runway) towards The Airport, which is located exactly 8 miles due north of the plane's current position. Flight 501 is scheduled to land at The Airport at 8:00 am, but, at 7:57 am, the control tower radios the plane and changes the landing location to an airport 15 miles directly due east of The Airport. Assuming a direct approach (and negligible time to shift direction), by how many miles per hour does the pilot have to increase her speed in order to arrive at the new location on time?
3. What is the ratio of the surface area of a cube to the surface area of a rectangular solid identical to the cube in all ways except that its length has been doubled?  
 $\frac{1}{4}$        $\frac{3}{8}$        $\frac{1}{2}$        $\frac{3}{5}$       2
4. A sphere is inscribed in a cube with an edge of 10. What is the shortest possible distance from one of the vertices of the cube to the surface of the sphere?  
 $10(\sqrt{3} - 1)$       5       $10(\sqrt{2} - 1)$        $5(\sqrt{3} - 1)$        $5(\sqrt{2} - 1)$
5. If the box shown is a cube, then the difference in length between line segment  $BC$  and line segment  $AB$  is approximately what fraction of the distance from  $A$  to  $C$ ?



10%


20%

30%

40%

50%

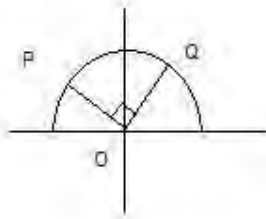
## Topic 7: Cylinders

1. A cylindrical tank of radius  $R$  and height  $H$  must be redesigned to hold approximately twice as much liquid. Which of the following changes would be farthest from the new design requirements?  
 a 100% increase in  $R$  and a 50% decrease in  $H$                       a 30% decrease in  $R$  and a 300% increase in  $H$   
 a 10% decrease in  $R$  and a 150% increase in  $H$                       a 40% increase in  $R$  and no change in  $H$   
 a 50% increase in  $R$  and a 20% decrease in  $H$
2. Cylinder A, which has a radius of  $x$  and a height of  $y$ , has a greater surface area than does Cylinder B, which has a radius of  $y$  and a height of  $x$ . How much greater is the surface area of Cylinder A than that of Cylinder of B?
3. A right circular cylinder has a radius  $r$  and a height  $h$ . What is the surface area of the cylinder?  
 (1)  $r = 2h - 2/h$                       (2)  $h = 15/r - r$
4. A cylindrical tank, with radius and height both of 10 feet, is to be redesigned as a cone, capable of holding twice the volume of the cylindrical tank. There are two proposed scenarios for the new cone: in scenario (1) the radius will remain the same as that of the original cylindrical tank, in scenario (2) the height will remain the same as that of the original cylindrical tank. What is the approximate difference in feet between the new height of the cone in scenario (1) and the new radius of the cone in scenario (2)?  
 (A) 13                      (B) 25                      (C) 30                      (D) 35                      (E) 40
5. The figure represents a deflated tire (6 inches wide as shown) with a hub (the center circle). The area of the hub surface shown in the picture is  $1/3$  the area of the tire surface shown in the picture. The thickness of the tire, when fully inflated is 3 inches. (Assume the tire material itself has negligible thickness.) Air is filled into the deflated tire at a rate of  $4\pi$  inches<sup>3</sup> / second. How long (in seconds) will it take to inflate the tire?  
  
 24                      27                      48                      81                      108
6. The contents of one full cylindrical silo are to be transferred to another, larger cylindrical silo. The contents of the smaller silo will fill what portion of the larger silo?  
 (1) The larger silo has twice the base radius, and twice the height, of the smaller one.  
 (2) The smaller silo has a circular base with a radius of 10 feet.
7. When a cylindrical tank is filled with water at a rate of 22 cubic meters per hour, the level of water in the tank rises at a rate of 0.7 meters per hour. Which of the following best approximates the radius of the tank in meters?  
 $\sqrt{10}/2$                        $\sqrt{10}$                       4                      5                      10
8. A 10-by-6 inch piece of paper is used to form the lateral surface of a cylinder. If the entire piece of paper is used to make the cylinder, which of the following must be true of the two possible cylinders that can be formed?
  - The volume of the cylinder with height 10 is 60/ cubic inches greater than the volume of the cylinder with height 6.
  - The volume of the cylinder with height 6 is 60/ cubic inches greater than the volume of the cylinder with height 10.
  - The volume of the cylinder with height 10 is 60 cubic inches greater than the volume of the cylinder with height 6.
  - The volume of the cylinder with height 6 is 60 cubic inches greater than the volume of the cylinder with height 10.
  - The volume of the cylinder with height 6 is 240/ cubic inches greater than the volume of the cylinder with height 10.

## GMAT Quant Topic 6

### Co-ordinate Geometry

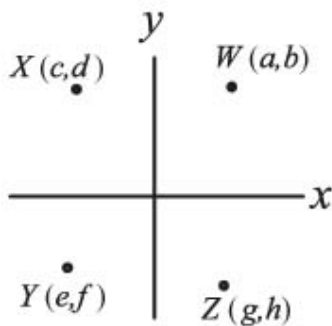
1. If  $ab \neq 0$  and points  $(-a,b)$  and  $(-b,a)$  are in the same quadrant of the  $xy$ -plane, is point  $(-x,y)$  in this same quadrant?  
(1)  $xy > 0$  (2)  $ax > 0$
2. In the  $xy$ -plane, at what two points does the graph of  $y = (x+a)(x+b)$  intersect the  $x$ -axis?  
(1)  $a + b = -1$  (2) The graph intersects the  $y$ -axis at  $(0, -6)$ .
3. For any triangle  $T$  in the  $xy$ -coordinate plan, the center of  $T$  is defined to be the point whose  $x$ -coordinate is the average (arithmetic mean) of the  $x$ -coordinates of the vertices of  $T$  and whose  $y$ -coordinate is the average of the  $y$ -coordinates of the vertices of  $T$ . If a certain triangle has vertices at the points  $(0,0)$  and  $(6,0)$  and center at the point  $(3,2)$ , what are the coordinates of the remaining vertex?  
A.  $(3,4)$  B.  $(3,6)$  C.  $(4,9)$  D.  $(6,4)$  E.  $(9,6)$
4. Circle  $C$  and line  $k$  lie in the  $xy$ -plane. if circle  $C$  is centered at the origin and has radius 1, does line  $k$  intersect circle  $C$ ?  
(1) the  $x$ -intercept of line  $k$  is greater than 1  
(2) the slope of line  $k$  is  $-1/10$
5. In the rectangular coordinate system, are the points  $(r,s)$  and  $(u,v)$  equidistant from the origin?  
(1)  $r + s = 1$  (2)  $u = 1 - r$  and  $v = 1 - s$
6. In the  $x$ - $y$  plane, what is the  $y$ -intercept of the line  $l$ ?  
(1) The slope of the line  $l$  is 3 times its  $y$  intercept.  
(2) The  $x$ -intercept of line  $l$  is  $-1/3$
7. In the figure shown, point  $P(-\sqrt{3}, 1)$  and  $Q(s, t)$  lie on the circle with center  $O$ .



What is value of  $s$ ?

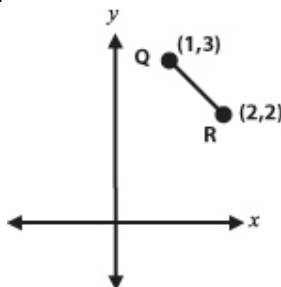
8. In the  $xy$ -plane, line  $k$  has positive slope and  $x$ -intercept 4. If the area of the triangle formed by line  $k$  and the two axes is 12, what of the  $y$ - intercept of line?
9. Line  $l$  is defined by the equation  $y - 5x = 4$  and line  $w$  is defined by the equation  $10y + 2x + 20 = 0$ . If line  $k$  does not intersect line  $l$ , what is the degree measure of the angle formed by line  $k$  and line  $w$ ?  
0      30      60      90      It cannot be determined from the information given.

10. In the rectangular coordinate plane points  $X$  and  $Z$  lie on the same line through the origin and points  $W$  and  $Y$  lie on the same line through the origin. If  $a^2 + b^2 = c^2 + d^2$  and  $e^2 + f^2 = g^2 + h^2$ , what is the value of length  $XZ$  – length  $WY$ ?



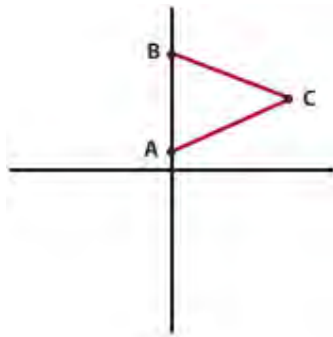
-2                      -1                      0                      1                      2

11. In the  $xy$ -coordinate system, what is the slope of the line that goes through the origin and is equidistant from the two points  $P = (1, 11)$  and  $Q = (7, 7)$ ?
- 2                      2.25                      2.50                      2.75                      3
12. What is the slope of the line represented by the equation  $x + 2y = 1$ ?
- $-3/2$      $-1$      $-1/2$      $0$      $1/2$
13. A certain square is to be drawn on a coordinate plane. One of the vertices must be on the origin, and the square is to have an area of 100. If all coordinates of the vertices must be integers, how many different ways can this square be drawn?
- 4                      6                      8                      10                      12
14. Does the equation  $y = (x - p)(x - q)$  intercept the  $x$ -axis at the point  $(2, 0)$ ?
- (1)  $pq = -8$                       (2)  $-2 - p = q$
15. Does line  $S$  intersect line segment  $QR$ ?



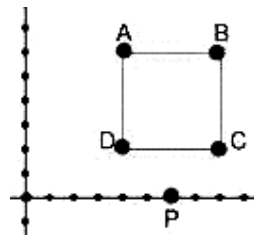
(1) The equation of line  $S$  is  $y = -x + 4$ . (2) The slope of line  $S$  is  $-1$ .

16. Line  $L$  contains the points  $(2, 3)$  and  $(p, q)$ . If  $q = 2$ , which of the following could be the equation of line  $m$ , which is perpendicular to line  $L$ ?
- (A)  $2x + y = px + 7$                       (B)  $2x + y = -px$                       (C)  $x + 2y = px + 7$   
 (D)  $y - 7 = x \div (p - 2)$                       (E)  $2x + y = 7 - px$
17. Point  $K = (A, 0)$ , Point  $G = (2A + 4, \sqrt{2A + 9})$ . Is the distance between point  $K$  and  $G$  prime?
- (1)  $A^2 - 5A - 6 = 0$                       (2)  $A > 2$
18. The  $(x, y)$  coordinates of points  $P$  and  $Q$  are  $(-2, 9)$  and  $(-7, -3)$ , respectively. The height of equilateral triangle  $XYZ$  is the same as the length of line segment  $PQ$ . What is the area of triangle  $XYZ$ ?
- $169/\sqrt{3}$                       84.5     $75\sqrt{3}$      $169\sqrt{3}/4$                        $225\sqrt{3}/4$
19. If points  $A$  and  $B$  are on the  $y$ -axis in the figure, what is the area of equilateral triangle  $ABC$ ?



- (1) The coordinates of point  $B$  are  $(0, 5\sqrt{3})$ .  
 (2) The coordinates of point  $C$  are  $(6, 3\sqrt{3})$ .

20. The line  $3x + 4y = 8$  passes through all of the quadrants in the coordinate plane except:  
 I          II          III          IV          II and IV.
21. If  $p$  and  $q$  are nonzero numbers, and  $p$  is not equal to  $q$ , in which quadrant of the coordinate system does point  $(p, p - q)$  lie?  
 (1)  $(p, q)$  lies in quadrant IV.          (2)  $(q, -p)$  lies in quadrant III.
22. The coordinates of points  $A$  and  $C$  are  $(0, -3)$  and  $(3, 3)$ , respectively. If point  $B$  lies on line  $AC$  between points  $A$  and  $C$ , and if  $AB = 2BC$ , which of the following represents the coordinates of point  $B$ ?  
 $(1, -\sqrt{5})$            $(1, -1)$            $(2, 1)$            $(1.5, 0)$            $(\sqrt{5}, \sqrt{5})$
23. In the  $xy$ -coordinate system, rectangle  $ABCD$  is inscribed within a circle having the equation  $x^2 + y^2 = 25$ . Line segment  $AC$  is a diagonal of the rectangle and lies on the  $x$ -axis. Vertex  $B$  lies in quadrant II and vertex  $D$  lies in quadrant IV. If side  $BC$  lies on line  $y = 3x + 15$ , what is the area of rectangle  $ABCD$ ?  
 (A) 15          (B) 30          (C) 40          (D) 45          (E) 50
24. The line represented by the equation  $y = 4 - 2x$  is the perpendicular bisector of line segment  $RP$ . If  $R$  has the coordinates  $(4, 1)$ , what are the coordinates of point  $P$ ?  
 (A)  $(-4, 1)$           (B)  $(-2, 2)$           (C)  $(0, 1)$           (D)  $(0, -1)$           (E)  $(2, 0)$
25. A certain computer program randomly generates equations of lines in the form  $y = mx + b$ . If point  $P$  is a point on a line generated by this program, what is the probability that the line does NOT pass through figure  $ABCD$ ?



- (A)  $\frac{3}{4}$           (B)  $\frac{3}{5}$           (C)  $\frac{1}{2}$           (D)  $\frac{2}{5}$           (E)  $\frac{1}{4}$

26. In the rectangular coordinate system, a line passes through the points  $(0, 5)$  and  $(7, 0)$ . Which of the following points must the line also pass through?  
 $(-14, 10)$            $(-7, 5)$            $(12, -4)$            $(14, -5)$            $(21, -9)$
27. Which of the following equations represents a line that is perpendicular to the line described by the equation  $3x + 4y = 8$ ?  
 $3x + 4y = 18$            $3x - 4y = 24$            $4y - 3x = 26$   
 $1.5y + 2x = 18$            $8x - 6y = 24$
28. How many units long is the straight line segment that connects the points  $(-1, 1)$  and  $(2, 6)$  on a rectangular coordinate plane?  
 4           $\sqrt{26}$            $\sqrt{34}$           7           $\sqrt{58}$
29. In the rectangular coordinate system, lines  $m$  and  $n$  cross at the origin. Is line  $m$  perpendicular to line  $n$ ?  
 (1)  $m$  has a slope of  $-1$  and  $n$  passes through the point  $(-a, -a)$ .

(2) If the slope of  $m$  is  $x$  and the slope of  $n$  is  $y$ , then  $-xy = 1$ .

30. Line  $A$  is drawn on a rectangular coordinate plane. If the coordinate pairs  $(3, 2)$  and  $(-1, -2)$  lie on line  $A$ , which of the following coordinate pairs does NOT lie on a line that is perpendicular to line  $A$ ?
- $(5, 8)$  and  $(4, 9)$        $(3, -1)$  and  $(4, -2)$        $(-1, 6)$  and  $(-4, 9)$   
 $(2, 5)$  and  $(-3, 2)$        $(7, 1)$  and  $(6, 2)$
31. Draw the following graphs (approximate shape)
- a.  $x^2 + 3x - 4 = 0$
  - b.  $2x^2 - 4x - 3 = 0$
  - c.  $x(x - 2) = 4$
  - d.  $9x^2 + 12x + 4 = 0$
  - e.  $3x^2 + 4x + 2 = 0$
  - f.  $x^2 + 2x = 1$
  - g.  $-2x^2 + 3x + 2$
  - h.  $2x^2 + 3x + 2$

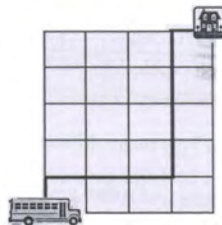
## GMAT Quant Topic 7

### Permutations and Combinations

1. How many different anagrams can you make for the word GMAT? How many different anagrams can you make for the word MATHEMATICS?
2. If there are 7 people and only 4 chairs in a room, how many different seating arrangements are possible?
3. A man wants to visit at least two of the four cities A, B, C and D. How many travel itineraries can he make? All cities are connected to one another.
4. There are 2 black balls, one red ball and one green ball, identical in shape and size. How many different linear arrangements can be generated by arranging these balls?
5. From a list of 10 songs, a DJ has to play either 2 or 3 songs. What is the total number of song sequences that he can create?
6. A password contains at least 8 distinct digits. It takes 12 seconds to try one combination, what is the minimum amount of time required to guarantee access to the database?
7. Greg, Marcia, Peter, Jan, Bobby and Cindy go to a movie and sit next to each other in 6 adjacent seats in the front row of the theater. If Marcia and Jan will not sit next to each other, in how many ways different arrangements can the 6 people sit?
8. If a team of 4 people is to be chosen from 7 people in a room, how many different teams are possible?
9. In a college, 8 students play at the State level and 10 at the National level. If 6 students play at both National and State levels, in how many ways can 9 students be selected from among these?
10. An engagement team consists of a project manager, team leader, and four consultants. There are 2 candidates for the position of project manager, 3 candidates for the position of team leader, and 7 candidates for the 4 consultant slots. If 2 out of 7 consultants refuse to be on the same team, how many different teams are possible?
11. In how many ways can 3 letters out of five distinct 5 distinct letters A, B, C, D and E be arranged in a straight line so that A and B never come together?
12. A nickel, a dime, and two identical quarters are arranged along a side of a table. If the quarters and the dime have to face heads up, while the nickel can face either heads up or tails up, how many different arrangements of coins are possible?
13. At a certain laboratory, chemical substances are identified by an unordered combination of 3 colors. If no chemical may be assigned the same colors, what is the maximum number of substances that can be identified using 7 colors?
14. An equity analyst needs to select 3 stocks for the upcoming year and rank these securities in terms of their investment potential. If the analyst has narrowed down the list of potential stocks to 7, in how many ways can she choose and rank her top 3 picks?
15. How many different five-letter combinations can be created from the word TWIST?
16. If an employee ID code must consist of 3 non-repeating digits and each digit in the code must be a prime number, how many ID codes can be created?
17. A university cafeteria offers 4 flavors of pizza – pepperoni, chicken, Hawaiian and vegetarian. If a customer has an option to add, extra cheese, mushrooms, or both to any kind of pizza, how many different pizza varieties are available?
18. Mario's Pizza has two choices of crust: deep dish and thin-and-crispy. The restaurant also has a choice of 5 toppings: tomatoes, sausage, peppers, onions, and pepperoni. Finally, Mario's offers every pizza in extra-cheese as well as 'regular'. If Linda's volleyball team decides to order a pizza with four toppings, how many different choices do the teammates have at Mario's Pizza?
19. A book store has received 8 different books, of which  $\frac{3}{8}$  are novels, 25% are study guides and the remaining are textbooks. If all books must be placed on one shelf displaying new items and if books in the same category have to be shelved next to each other, how many different arrangements of books are possible?



20. A group of 5 students bought movie tickets in one row next to each other. If Bob and Lisa are in this group, what is the number of ways of seating if both of them will sit next to only one other student from the group?
21. Mark's clothing store uses a bar-code system to identify every item. Each item is marked by a combination of 2 letters followed by 3 digits. Additionally, the three-digit number must be even for male products and odd for female products. If all apparel products start with the letter combination AP, how many male apparel items can be identified with the bar code?
22. Fernando purchased a university meal plan that allows him to have a total of 3 lunches and 3 dinners per week. If the cafeteria is closed on weekends and Fernando always goes home for a dinner on Friday nights, how many options does he have to allocate his meals?
23. If the President and the Vice President must sit next to each other in a row with 4 other members on the Board, how many different seating arrangements are possible?
24. To apply for the position of photographer at a local magazine, Veronica needs to include 3 or 4 of her pictures in an envelope accompanying her application. If she has pre-selected 5 photos representative of her work, how many choices does she have to provide the photos for the magazine?
25. A retail company needs to set up 5 additional distribution centers that can be located in three cities on the east coast (Boston, New York, and Washington D.C.), one city in the mid-west (Chicago), and three cities on the west coast (Seattle, San Francisco and Los Angeles). If the company must have 2 distribution centers on each coast and 1 in the mid-west, and only one center can be added in each city, in how many ways can the management allocate the distribution centers?
26. Three couples need to be arranged in a row for a group photo. If the couples cannot be separated, how many different arrangements are possible?
27. If 6 fair coins are tossed, how many different coin sequences will have exactly 3 tails, if all tails have to occur in a row?
28. A telephone company needs to create a set of 3-digit area codes. The company is entitled to use only digits 2, 4 and 5, which can be repeated. If the product of the digits in the area code must be even, how many different codes can be created?
29. Jake, Lena, Fred, John and Inna need to drive home from a corporate reception in an SUV that can seat 7 people. If only Inna or Jake can drive, how many seat allocations are possible?
30. In how many ways can a teacher write an answer key for a mini-quiz that contains 3 true-false questions followed by 2 multiples-choice questions with 4 answer choices each, if the correct answers to all true-false questions cannot be the same?
31. A student committee on academic integrity has 90 ways to select a president and vice-president from a group of candidates. The same person cannot be both president and vice-president. How many students are in the group?
32. A pod of 6 dolphins always swims single file, with 3 females at the front and 3 males in the rear. In how many different arrangements can the dolphins swim?
33. A British spy is trying to escape from his prison cell. The lock requires him to enter one number, from 1-9, and then push a pair of colored buttons simultaneously. He can make one attempt every 3 seconds. If there are 6 colored buttons, what is the longest possible time it could take the spy to escape from the prison cell?
34. Every morning, Casey walks from her house to the bus stop. She always travels exactly nine blocks from her house to the bus, but she varies the route she takes every day. (One sample route is shown.) How many days can Casey walk from her house to the bus stop without repeating the same route?



35. Three dwarves and three elves sit down in a row of six chairs. If no dwarf will sit next to another dwarf and no elf will sit next to another elf, in how many different ways can the elves and dwarves sit?

36. Gordon buys 5 dolls for his 5 nieces. The gifts include two identical Sun-and-Fun beach dolls, one Elegant Eddie dress-up doll, one G.I. Josie army doll, and one Tulip Troll doll. If the youngest niece doesn't want the G.I. Josie doll, in how many different ways can he give the gifts?
37. How many different 5-person teams can be formed from a group of  $x$  individuals?
- (1) If there had been  $x + 2$  individuals in the group, exactly 126 different 5-person teams could have been formed.
  - (2) If there had been  $x + 1$  individuals in the group, exactly 56 different 3-person teams could have been formed.
38. There are  $x$  people and  $y$  chairs in a room where  $x$  and  $y$  are positive prime numbers. How many ways can the  $x$  people be seated in the  $y$  chairs (assuming that each chair can seat exactly one person)?
- (1)  $x + y = 12$
  - (2) There are more chairs than people.
39. A gambler began playing blackjack with \$110 in chips. After exactly 12 hands, he left the table with \$320 in chips, having won some hands and lost others. Each win earned \$100 and each loss cost \$10. How many possible outcomes were there for the first 5 hands he played? (For example, won the first hand, lost the second, etc.)
- (A) 10                      (B) 18                      (C) 26                      (D) 32                      (E) 64
40. In a 4 person race, medals are awarded to the fastest 3 runners. The first-place runner receives a gold medal, the second-place runner receives a silver medal, and the third-place runner receives a bronze medal. In the event of a tie, the tied runners receive the same color medal. (For example, if there is a two-way tie for first-place, the top two runners receive gold medals, the next-fastest runner receives a silver medal, and no bronze medal is awarded). Assuming that exactly three medals are awarded, and that the three medal winners stand together with their medals to form a victory circle, how many different victory circles are possible?
- (A) 24                      (B) 52                      (C) 96                      (D) 144                      (E) 648
41. The organizers of a week-long fair have hired exactly five security guards to patrol the fairgrounds at night for the duration of the event. Exactly two guards are assigned to patrol the grounds every night, with no guard assigned consecutive nights. If the fair begins on a Monday, how many different pairs of guards will be available to patrol the fairgrounds on the following Saturday night?
- (A) 9                      (B) 7                      (C) 5                      (D) 3                      (E) 2
42. Larry, Michael, and Doug have five donuts to share. If any one of the men can be given any whole number of donuts from 0 to 5, in how many different ways can the donuts be distributed?
- (A) 21                      (B) 42                      (C) 120                      (D) 504                      (E) 5040
43. A woman has seven cookies—four chocolate chip and three oatmeal. She gives one cookie to each of her six children: Nicole, Ronit, Kim, Deborah, Mark, and Terrance. If Deborah will only eat the kind of cookie that Kim eats, in how many different ways can the cookies be distributed? (The leftover cookie will be given to the dog.)
- (A) 5040                      (B) 50                      (C) 25                      (D) 15                      (E) 12
44. Sammy has  $x$  flavors of candies with which to make goody bags for Frank's birthday party. Sammy tosses out  $y$  flavors, because he doesn't like them. How many different 10-flavor bags can Sammy make from the remaining flavors? (It doesn't matter how many candies are in a bag, only how many flavors).
- (1) If Sammy had thrown away 2 additional flavors of candy, he could have made exactly 3,003 different 10-flavor bags.
  - (2)  $x = y + 17$
45. How many different combinations of outcomes can you make by rolling three standard (6-sided) dice if the order of the dice does not matter?
- (A) 24                      (B) 30                      (C) 56                      (D) 120                      (E) 216
46. A certain league has four divisions. The respective divisions had 9, 10, 11, and 12 teams qualify for the playoffs. Each division held its own double-elimination tournament -- where a team is eliminated from the tournament upon losing two games -- in order to determine its champion. The four division champions then played in a single-elimination tournament -- where a team is eliminated upon losing one game -- in order to

determine the overall league champion. Assuming that there were no ties and no forfeits, what is the maximum number of games that could have been played in order to determine the overall league champion?

- (A) 79                      (B) 83                      (C) 85                      (D) 87                      (E) 88

47. You have a bag of 9 letters: 3 Xs, 3 Ys, and 3 Zs. You are given a box divided into 3 rows and 3 columns for a total of 9 areas. How many different ways can you place one letter into each area such that there are no rows or columns with 2 or more of the same letter? (Note: One such way is shown below.)

X	Y	Z
Y	Z	X
Z	X	Y

- (A) 5                      (B) 6                      (C) 9                      (D) 12                      (E) 18

48. Eight women of eight different heights are to pose for a photo in two rows of four. Each woman in the second row must stand directly behind a shorter woman in the first row. In addition, all of the women in each row must be arranged in order of increasing height from left to right. Assuming that these restrictions are fully adhered to, in how many different ways can the women pose?

- (A) 2                      (B) 14                      (C) 15                      (D) 16                      (E) 18

49. Company X has 6 regional offices. Each regional office must recommend two candidates, one male and one female, to serve on the corporate auditing committee. If each of the offices must be represented by exactly one member on the auditing committee and if the committee must consist of an equal number of male and female employees, how many different committees can be formed?

50. You have a six-sided cube and six cans of paint, each a different color. You may not mix colors of paint. How many distinct ways can you paint the cube using a different color for each side? (If you can reorient a cube to look like another cube, then the two cubes are not distinct.)

- (A) 24                      (B) 30                      (C) 48                      (D) 60                      (E) 120

51. A group of four women and three men have tickets for seven adjacent seats in one row of a theatre. If the three men will not sit in three adjacent seats, how many possible different seating arrangements are there for these 7 theatre-goers?

- (A)  $7! - 2!3!2!$     (B)  $7! - 4!3!$     (C)  $7! - 5!3!$     (D)  $7 \times 2!3!2!$     (E)  $2!3!2!$

52. Anthony and Michael sit on the six-member board of directors for company X. If the board is to be split up into 2 three-person subcommittees, what percent of all the *possible* subcommittees that include Michael also include Anthony?

- 20%                      30%                      40%                      50%                      60%

53. A family consisting of one mother, one father, two daughters and a son is taking a road trip in a sedan. The sedan has two front seats and three back seats. If one of the parents must drive and the two daughters refuse to sit next to each other, how many possible seating arrangements are there?

- 28                      32                      48                      60                      120

54. Six mobsters have arrived at the theater for the premiere of the film "Goodbuddies." One of the mobsters, Frankie, is an informer, and he's afraid that another member of his crew, Joey, is on to him. Frankie, wanting to keep Joey in his sights, insists upon standing behind Joey in line at the concession stand. How many ways can the six arrange themselves in line such that Frankie's requirement is satisfied?

- 6                      24                      120                      360                      720

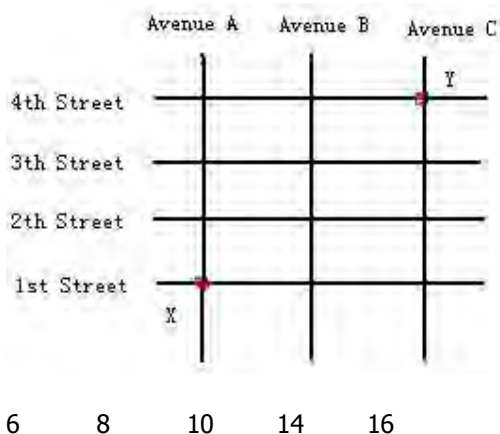
55. A college admissions committee will grant a certain number of \$10,000 scholarships, \$5,000 scholarships, and \$1,000 scholarships. If no student can receive more than one scholarship, how many different ways can the committee dole out the scholarships among the pool of 10 applicants?

- (1) In total, six scholarships will be granted.  
(2) An equal number of scholarships will be granted at each scholarship level.

56. A certain panel is to be composed of exactly three women and exactly two men, chosen from  $x$  women and  $y$  men. How many different panels can be formed with these constraints?

- (1) If two more women were available for selection, exactly 56 different groups of three women could be selected.  
(2)  $x = y + 1$

57. A student committee that must consist of 5 members is to be formed from a pool of 8 candidates. How many different committees are possible?  
 5                      8                      40                      56                      336
58. How many ways are there to award a gold, silver and bronze medal to 10 contending teams?  
 $10 \times 9 \times 8$                        $10! / 3! 7!$                        $10! / 3!$                       360                      300
59. From a drawer containing black, blue and gray solid-color socks, including at least three socks of each color, how many matched pairs can be removed?  
 (1) The drawer contains 11 socks.  
 (2) The drawer contains an equal number of black and gray socks.
60. On Tuesday, Kramer purchases exactly 3 new shirts, 2 new sweaters, and 4 new hats, On the following day and each subsequent day thereafter, Kramer wears one of his new shirts together with one of his new sweaters and one of his new hats. Kramer avoids wearing the exact same combination of shirt, sweater, and hat for as long as possible. On which day is this no longer possible?  
 Tuesday                      Wednesday                      Thursday                      Friday                      Saturday
61. A certain stock exchange designates each stock with a one-, two-, or three-letter code, where each letter is selected from the 26 letters of the alphabet. If the letter may be repeated and if the same letters used in a different order constitute a different code, how many different stocks is it possible to uniquely designate with these codes?  
 2951                      8125                      15600                      15302                      18278
62. A certain law firm consists of 4 senior partners and 6 junior partners. How many different groups of 3 partners can be formed in which at least one member of the group is a senior partner? (Two groups are considered different if at least one group member is different.)  
 48                      100                      120                      288                      600
63. A company plans to assign identification numbers to its employees. Each number is to consist of four different digits from 0 to 9, inclusive, except that the first digit cannot be 0. How many different identification numbers are possible?
64. Pat will walk from intersection X to intersection Y along route that is confined to the square grid of four streets and three avenues shown in the map above. How many routes from X to Y can Pat take that have the minimum possible length?



65. Tanya prepared four different letters to be sent to four different addresses. For each letter, she prepared an envelope with its correct address. If the 4 letters are to be put into the envelopes at random, what is the probability that only one letter will be put into the envelope with its correct address?
66. 5 people are to be seated around a circular table. Two seating arrangement are considered different only when the positions of the people are different relative to each other. What is the total number of different possible seating arrangements for the group?



1.  $4!, 11! / 2! * 2! * 2!$
2.  ${}^7P_4$
3.  ${}^4P_2 + {}^4P_3 + {}^4P_4$
4.  $4! / 2!$
5.  ${}^{10}P_2 + {}^{10}P_3$
6.  $({}^{10}P_8 + {}^{10}P_9 + {}^{10}P_{10}) \times 12 \text{ seconds}$
7.  $6! - 5! \times 2!$
8.  ${}^7C_4$
9.  ${}^{12}C_9$
10.  ${}^2C_1 \times {}^3C_1 \times ({}^7C_4 - {}^5C_2)$
11.  $({}^5C_3 - {}^3C_1) \times 3!$
12.  $(4! / 2!) \times 2!$
13.  ${}^7C_3$
14.  ${}^7P_3$
15.  $5! / 2!$
16.  ${}^4P_3$
17.  $4 \times 4 = 16$
18. Another version:  $2 \times {}^5C_4 \times 2 = 20$
19.  $(3! \times 2! \times 3!) \times 3!$
20.  $3! \times 2!$
21.  $1 \times 10 \times 10 \times 5$
22.  ${}^5C_3 \times {}^4C_3$
23.  $5! \times 2!$
24.  ${}^5C_3 + {}^5C_4$
25.  ${}^3C_2 \times 1 \times {}^3C_2$
26.  $3! \times 2! \times 2! \times 2!$
27. TTTTHH, HTTTHH, HHTTTH, HHHTTT; total  
4
28. The only combination of odd is  $5 \times 5 \times 5$ .  
So total required =  $3 \times 3 \times 3 - 1 = 26$ .
29.  ${}^6P_4 \times 2 = 720$
30.  $2 \times 2 \times 2 \times 4 \times 4 - 2 \times 1 \times 1 \times 4 \times 4 =$   
96
31. 10
32.  $3! \times 3! = 36$
33.  ${}^6C_2 \times 9 \times 3 = 405 \text{ seconds}$
34.  $9! / 5! \times 4!$
35.  $6 \times 2 \times 1 \times 3 \times 2 \times 1 = 72$
36.  $(5! / 2! - 4! / 2!) = 48$ .
37. D
38. A
39. C
40. B
41. D
42. A
43. D
44. D
45. C
46. B
47. D
48. B
49. 20
50. B
51. B
52. C
53. B
54. D
55. C
56. C
57. D
58. A
59. E
60. E
61. E
62. B
63. 4536
64. C
65.  $1/3$
66. 24

## GMAT Quant Topic 8

### Probability

67. A fair coin is flipped three times. What is the probability that the coin lands on heads exactly twice?  
(A)  $\frac{1}{8}$  (B)  $\frac{3}{8}$  (C)  $\frac{1}{2}$  (D)  $\frac{5}{8}$  (E)  $\frac{7}{8}$
68. Is the probability that Patty will answer all of the questions on her chemistry exam correctly greater than 50%?  
(1) For each question on the chemistry exam, Patty has a 90% chance of answering the question correctly.  
(2) There are fewer than 10 questions on Patty's chemistry exam.
69. There are 10 women and 3 men in room A. One person is picked at random from room A and moved to room B, where there are already 3 women and 5 men. If a single person is then to be picked from room B, what is the probability that a woman will be picked?  
(A)  $\frac{13}{21}$  (B)  $\frac{49}{117}$  (C)  $\frac{15}{52}$  (D)  $\frac{5}{18}$  (E)  $\frac{40}{117}$
70. If the probability of rain on any given day in Chicago during the summer is 50%, independent of what happens on any other day, what is the probability of having exactly 3 rainy days from July 4 through July 8, inclusive?  
(A)  $\frac{1}{32}$  (B)  $\frac{2}{25}$  (C)  $\frac{5}{16}$  (D)  $\frac{8}{25}$  (E)  $\frac{3}{4}$
71. In a shipment of 20 cars, 3 are found to be defective. If four cars are selected at random, what is the probability that exactly one of the four will be defective?  
(A)  $\frac{170}{1615}$  (B)  $\frac{3}{20}$  (C)  $\frac{8}{19}$  (D)  $\frac{3}{5}$  (E)  $\frac{4}{5}$
72. A certain bag of gemstones is composed of two-thirds diamonds and one-third rubies. If the probability of randomly selecting two diamonds from the bag, without replacement, is  $\frac{5}{12}$ , what is the probability of selecting two rubies from the bag, without replacement?  
(A)  $\frac{5}{36}$  (B)  $\frac{5}{24}$  (C)  $\frac{1}{12}$  (D)  $\frac{1}{6}$  (E)  $\frac{1}{4}$
73. Triplets Adam, Bruce, and Charlie enter a triathlon. If there are 9 competitors in the triathlon and medals are awarded for first, second, and third place, what is the probability that at least two of the triplets will win a medal?  
(A)  $\frac{3}{14}$  (B)  $\frac{19}{84}$  (C)  $\frac{11}{42}$  (D)  $\frac{15}{28}$  (E)  $\frac{3}{4}$
74. Set  $S$  is the set of all prime integers between 0 and 20. If three numbers are chosen randomly from set  $S$  and each number can be chosen only once, what is the positive difference between the probability that the product of these three numbers is a number less than 31 and the probability that the sum of these three numbers is odd?  
(A)  $\frac{1}{336}$  (B)  $\frac{1}{2}$  (C)  $\frac{17}{28}$  (D)  $\frac{3}{4}$  (E)  $\frac{301}{336}$
75. A random 10-letter code is to be formed using the letters A, B, C, D, E, F, G, H, I and I (only the "I" will be used twice). What is the probability that a code that has the two I's adjacent to one another will be formed?  
(A)  $\frac{1}{10}$  (B)  $\frac{1}{8}$  (C)  $\frac{1}{5}$  (D)  $\frac{1}{4}$  (E)  $\frac{1}{2}$
76. If  $p^2 - 13p + 40 = q$ , and  $p$  is a positive integer between 1 and 10, inclusive, what is the probability that  $q < 0$ ?  
(A)  $\frac{1}{10}$  (B)  $\frac{1}{5}$  (C)  $\frac{2}{5}$  (D)  $\frac{3}{5}$  (E)  $\frac{3}{10}$
77. A box contains three pairs of blue gloves and two pairs of green gloves. Each pair consists of a left-hand glove and a right-hand glove. Each of the gloves is separate from its mate and thoroughly mixed together with the others in the box. If three gloves are randomly selected from the box, what is the probability that a matched set (i.e., a left- and right-hand glove of the same color) will be among the three gloves selected?  
(A)  $\frac{3}{10}$  (B)  $\frac{23}{60}$  (C)  $\frac{7}{12}$  (D)  $\frac{41}{60}$  (E)  $\frac{5}{6}$
78. A football team has 99 players. Each player has a uniform number from 1 to 99 and no two players share the same number. When football practice ends, all the players run off the field one-by-one in a completely

random manner. What is the probability that the first four players off the field will leave in order of increasing uniform numbers (e.g., #2, then #6, then #67, then #72, etc)?

- (A)  $1/64$  (B)  $1/48$  (C)  $1/36$  (D)  $1/24$  (E)  $1/16$

79. What is the probability that  $\frac{u}{w}$  and  $\frac{x}{z}$  are reciprocal fractions?
- (1)  $u$ ,  $w$ ,  $y$ , and  $z$  are each randomly chosen from the first 100 positive integers.  
 (2) The product  $(u)(x)$  is the median of 100 consecutive integers.
80. Bill and Jane play a simple game involving two fair dice, each of which has six sides numbered from 1 to 6 (with an equal chance of landing on any side). Bill rolls the dice and his score is the total of the two dice. Jane then rolls the dice and her score is the total of her two dice. If Jane's score is higher than Bill's, she wins the game. What is the probability the Jane will win the game?  
 (A)  $15/36$  (B)  $175/432$  (C)  $575/1296$  (D)  $583/1296$  (E)  $1/2$
81. Kate and Danny each have \$10. Together, they flip a fair coin 5 times. Every time the coin lands on heads, Kate gives Danny \$1. Every time the coin lands on tails, Danny gives Kate \$1. After the five coin flips, what is the probability that Kate has more than \$10 but less than \$15?  
 (A)  $5/16$  (B)  $1/2$  (C)  $12/30$  (D)  $15/32$  (E)  $3/8$
82. There is a 10% chance that it won't snow all winter long. There is a 20% chance that schools will not be closed all winter long. What is the greatest possible probability that it will snow and schools will be closed during the winter?  
 (A) 55% (B) 60% (C) 70% (D) 72% (E) 80%
83. There are  $y$  different travelers who each have a choice of vacationing at one of  $n$  different destinations. What is the probability that all  $y$  travelers will end up vacationing at the same destination?  
 (A)  $1/n!$  (B)  $n/n!$  (C)  $1/n^y$  (D)  $1/n^{y-1}$  (E)  $n/y^n$
84. A small, experimental plane has three engines, one of which is redundant. That is, as long as two of the engines are working, the plane will stay in the air. Over the course of a typical flight, there is a  $1/3$  chance that engine one will fail. There is a 75% probability that engine two will work. The third engine works only half the time. What is the probability that the plane will crash in any given flight?  
 (A)  $7/12$  (B)  $1/4$  (C)  $1/2$  (D)  $7/24$  (E)  $17/24$
85. In the game of Funball, each batter can either hit a home run, hit a single, or strikeout, and the likelihood of each outcome is completely determined by the opposing pitcher. A Funball batter scores a point for their team by advancing sequentially through each of four "bases", according to the following rules:  
**Home run:** The batter and any players already on a base advance through all four bases.  
**Single:** The batter advances to first base, and any players already on a base advance one base each.  
**Strikeout:** No one advances any bases, and the batter loses his/her turn.  
 If the batting team has a runner on first base, which pitcher (Roger or Greg) is more likely to allow a point before recording a strikeout?  
 (1) Greg is twice as likely as Roger to allow a single, and four times as likely as Roger to record a strikeout.  
 (2) Greg is twice as likely as Roger to allow a single, and one fourth as likely as Roger to allow a home run.
86. Ms. Barton has four children. You are told correctly that she has at least two girls but you are not told which two of her four children are those girls. What is the probability that she also has two boys? (Assume that the probability of having a boy is the same as the probability of having a girl.)  
 (A)  $1/4$  (B)  $3/8$  (C)  $5/11$  (D)  $1/2$  (E)  $6/11$
87. Mike recently won a contest in which he will have the opportunity to shoot free throws in order to win \$10,000. In order to win the money Mike can either shoot 1 free throw and make it, or shoot 3 free throws and make at least 2 of them. Mike occasionally makes shots and occasionally misses shots. He knows that his probability of making a single free throw is  $p$ , and that this probability doesn't change. Would Mike have a better chance of winning if he chose to attempt 3 free throws?  
 (1)  $p < 0.7$  (2)  $p > 0.6$



88. Laura has a deck of standard playing cards with 13 of the 52 cards designated as a "heart." If Laura shuffles the deck thoroughly and then deals 10 cards off the top of the deck, what is the probability that the 10th card dealt is a heart?  
 (A)  $\frac{1}{4}$  (B)  $\frac{1}{5}$  (C)  $\frac{5}{26}$  (D)  $\frac{12}{42}$  (E)  $\frac{13}{42}$
89. A license plate in the country Kerrania consists of four digits followed by two letters. The letters A, B, and C are used only by government vehicles while the letters D through Z are used by non-government vehicles. Kerrania's intelligence agency has recently captured a message from the country Gonzalia indicating that an electronic transmitter has been installed in a Kerrania government vehicle with a license plate starting with 79. If it takes the police 10 minutes to inspect each vehicle, what is the probability that the police will find the transmitter within three hours?  
 (A)  $\frac{18}{79}$  (B)  $\frac{1}{6}$  (C)  $\frac{1}{25}$  (D)  $\frac{1}{50}$  (E)  $\frac{1}{900}$
90. A grid of light bulbs measures  $x$  bulbs by  $x$  bulbs, where  $x > 2$ . If 4 light bulbs are illuminated at random, what is the probability, in terms of  $x$ , that the 4 bulbs form a 2 bulb by 2 square?
91. Baseball's World Series matches 2 teams against each other in a best-of-seven series. The first team to win four games wins the series and no subsequent games are played. If you have no special information about either of the teams, what is the probability that the World Series will consist of fewer than 7 games?  
 (A) 12.5% (B) 25% (C) 31.25% (D) 68.75% (E) 75%
92. Harriet and Tran each have \$10. Together, they flip a fair coin 5 times. Every time the coin lands on heads, Tran gives Harriet \$1. Every time the coin lands on tails, Harriet gives Tran \$1. After the five coin flips, what is the probability that Harriet has more than \$10 but less than \$15?  
 (A)  $\frac{5}{16}$  (B)  $\frac{1}{2}$  (C)  $\frac{12}{30}$  (D)  $\frac{15}{32}$  (E)  $\frac{3}{8}$
93. If 40 percent of all students at College  $X$  have brown hair and 70 percent of all students at College  $X$  have blue eyes, what is the difference between the minimum and the maximum probability of picking a student from College  $X$  who has neither brown hair nor blue eyes?  
 (A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.7
94. In a room filled with 7 people, 4 people have exactly 1 friend in the room and 3 people have exactly 2 friends in the room (Assuming that friendship is a mutual relationship, i.e. if John is Peter's friend, Peter is John's friend). If two individuals are selected from the room at random, what is the probability that those two individuals are NOT friends?  
 $\frac{5}{21}$   $\frac{3}{7}$   $\frac{4}{7}$   $\frac{5}{7}$   $\frac{16}{21}$
95. Bill has a small deck of 12 playing cards made up of only 2 suits of 6 cards each. Each of the 6 cards within a suit has a different value from 1 to 6; thus, there are 2 cards in the deck that have the same value. Bill likes to play a game in which he shuffles the deck, turns over 4 cards, and looks for pairs of cards that have the same value. What is the chance that Bill finds at least one pair of cards that have the same value?  
 $\frac{8}{33}$   $\frac{62}{165}$   $\frac{17}{33}$   $\frac{103}{165}$   $\frac{25}{33}$
96. If a jury of 12 people is to be selected randomly from a pool of 15 potential jurors, and the jury pool consists of  $\frac{2}{3}$  men and  $\frac{1}{3}$  women, what is the probability that the jury will comprise at least  $\frac{2}{3}$  men?  
 $\frac{24}{91}$   $\frac{45}{91}$   $\frac{2}{3}$   $\frac{67}{91}$   $\frac{84}{91}$
97. John and Peter are among the nine players a basketball coach can choose from to field a five-player team. If all five players are chosen at random, what is the probability of choosing a team that includes John and Peter?  
 $\frac{1}{9}$   $\frac{1}{6}$   $\frac{2}{9}$   $\frac{5}{18}$   $\frac{1}{3}$
98. A small company employs 3 men and 5 women. If a team of 4 employees is to be randomly selected to organize the company retreat, what is the probability that the team will have exactly 2 women?  
 $\frac{1}{14}$   $\frac{1}{7}$   $\frac{2}{7}$   $\frac{3}{7}$   $\frac{1}{2}$
99. A box contains one dozen donuts. Four of the donuts are chocolate, four are glazed, and four are jelly. If two donuts are randomly selected from the box, one after the other, what is the probability that both will be jelly donuts?  
 $\frac{1}{11}$   $\frac{1}{9}$   $\frac{1}{3}$   $\frac{2}{3}$   $\frac{8}{9}$

100. 8 cities, including Memphis, compete in a national contest to host a political convention. Exactly one city wins the competition. What is the probability that Memphis does not win the competition?  
 (1) The probability that any one of the 8 cities does not win the competition is  $7/8$ .  
 (2) The probability that Memphis wins the competition is  $1/8$ .
101. A hand purse contains 6 nickels, 5 pennies and 4 dimes. What is the probability of picking a coin other than a nickel twice in a row if the first coin picked is not put back?  
 $8/25$     $12/35$     $13/35$     $9/25$     $17/25$
102. Jim and Renee will play one game of Rock, Paper, Scissors. In this game, each will select and show a hand sign for one of the three items. Rock beats Scissors, Scissors beat Paper, and Paper beats Rock. Assuming that both Jim and Renee have an equal chance of choosing any one of the hand signs, what is the probability that Jim will win?  
 $5/6$     $2/3$     $1/2$     $5/12$     $1/3$
103. A certain box contains only red balls and green balls. If one ball is randomly selected from the box, what is the probability that it is red?  
 (1) Red balls comprise exactly two-thirds of all the balls in the box.  
 (2) The probability of selecting a green ball from the box is  $1/3$ .
104. At a certain car dealership, the 40 vehicles equipped with air conditioning represent 80% of all cars available for sale. Among all the cars, there are 15 convertibles, 14 of which are equipped with an air-conditioning system. If a customer is willing to purchase either a convertible or a car equipped with air conditioning, what is the probability that a randomly selected vehicle will fit customer specifications?
105. In a card game, a combination of two aces beats all others. If Jose is the first to draw from a standard deck of 52 cards, what is the probability that he wins the game with the best possible combination?
106. Derrick and Lena, a married couple attending the same business school, go to a corporate presentation that ends with two sequential drawings of a PDA (Personal Digital Assistant) among the 60 attending students. If each attendant is given one ticket participating in the lottery of the two PDAs and if each winning ticket is removed from the urn, what is the probability that both PDAs will go to the couple?
107. If two balls are randomly drawn from a green urn containing 5 black and 5 white balls and placed into a yellow urn initially containing 5 black and 3 white balls, what is the probability that the yellow urn will contain an equal number of black and white balls after this change?
108. In a certain game of dice, the player's score is determined as a sum of three throws of a single die. The player with the highest score wins the round. If more than one player has the highest score, the winnings of the round are divided equally among these players. If Jim plays this game against 21 other players, what is the probability of the minimum score that will guarantee Jim some monetary payoff?
109. Mathematics, Physics and Chemistry books are stored on a library shelf that can accommodate 25 books. Currently 20% of the shelf spots remain empty. There are twice as many mathematics books as physics books and the number of physics books is 4 greater than that of the chemistry books. Ricardo selects one book at random from the shelf, reads it in the library, and then returns it to the shelf. Then he again chooses one book at random from the shelf and checks it out in order to read at home. What is the probability that Ricardo reads one book on mathematics and one on chemistry?
110. Maria bought 4 black and a certain number of red and blue pencils at 15 cents each and carried them home in one bag. After Maria came home, she took out one pencil at random to write a note for her friend and then put this pencil back into the bag. Some time later, she needed to write another note and again took out a pencil from the bag. If the probability that Maria wrote both her notes in black is  $1/36$ , how much did she spend on all pencils?
111. If Jessica tosses a coin 3 times, what is the probability that she will get heads at least once?
112. Set S consists of numbers 2, 3, 6, 48 and 164. Number K is computed by multiplying one random number from set S by one of the first 10 non-negative integers, also selected at random. If  $Z = 6^k$ , what is the probability that 678463 is not a multiple of Z?

113. Two identical urns—black and white—each contain 5 blue, 5 red and 10 green balls. Every ball selected from the black urn is immediately returned to the urn, while each ball selected from the white urn is removed and placed on a table. If Jenny receives a quarter for every blue ball, a dime for every red ball and a nickel for every green ball she selects, what is the probability that she will be able to buy a 25-cent candy bar with the proceeds from drawing four balls—two from each urn?
114. According to a recent student poll, 15 out of 21 members of the finance club are interested in a career in investment banking. If two students are chosen at random, what is the probability that at least one of them is interested in investment banking?
115. If 4 fair dice are thrown simultaneously, what is the probability of getting at least one pair?
116. Operation '#' is defined as adding a randomly selected two-digit multiple of 6 to a randomly selected two-digit prime number and reducing the result by half. If operation '#' is repeated 10 times, what is the probability that it will yield at least two integers?
117. Number N is randomly selected from a set of consecutive integers between 50 and 69, inclusive. What is the probability that N will have the same number of factors as 89?
118. Each year three space shuttles are launched, two in June and one in October. If each shuttle is known to occur without a delay in 90% of the cases and if the current month is January, what is the probability that at least one of the launches in the next 16 months will be delayed?
119. Rowan throws 3 dice and records the product of the numbers appearing at the top of each die as the result of the attempt. What is the probability that the result of any attempt is an odd integer divisible by 25?
120. A telephone number contains 10 digits, including a 3-digit area code. Bob remembers the area code and the next 5 digits of the number. He also remembers that the remaining digits are not 0, 1, 2, 5, or 7. If Bob tries to find the number by guessing the remaining digits at random, the find probability that he will be able to find the correct number in at most 2 attempts.
121. If number N is randomly drawn from a set of all non-negative single-digit integers, what is the probability that  $5N^3/8$  is an integer?
122. The acceptance rate at a certain business school is 15% for the first time applicants and 20% for all re-applicants. If David is applying for admission for the first time this year, what is the probability that he will have to apply no more than twice before he is accepted?
123. If a randomly selected positive single digit multiple of 3 is multiplied by a randomly selected prime number less than 20, what is the probability that this product will be a multiple of 45?
124. If a pencil is selected at random from a desk drawer, what is the probability that this pencil is red?  
 (1) There are 6 black and 4 orange pencils among the pencils in the drawer.  
 (2) There are three times as many red pencils in the drawer as pencils of all other colors combined.
125. What is the probability of selecting a white ball from an urn?  
 (1) There are twice as many white balls as there are balls of any other color.  
 (2) There are 30 more white balls as balls of all other colors combined.
126. Jonathan would like to visit one of the 12 gyms in his area. If he selects a gym at random, what is the probability that the gym will have both a swimming pool and a squash court?  
 (1) All but 2 gyms in the area have a squash court.  
 (2) Each of the 9 gyms with a pool has a squash court.
127. There were initially no black marbles in a jar. Subsequently, new marbles were added to the jar. If marbles are drawn at random and selected marbles are not returned to the jar, what is the probability of selecting 2 black marbles in a row?  
 (1) After the new marbles are added, 50% of all marbles are black.  
 (2) Among the 10 added marbles, 8 are black.
128. What is the probability that it will rain on each of the next 3 days if the probability of raining on any single day is the same in that period?  
 (1) The probability of no rain throughout the first two days is 36%.

(2) The probability of rain on the third day is 40%.

129. If a number is drawn at random from the first 1000 positive integers, what is the probability of selecting a refined number?  
 (1) Any refined number must be divisible by 22.  
 (2) A refined number is any even multiple of 11.
130. Number N is randomly selected from a set of all primes between 10 and 40, inclusive. Number K is selected from a set of all multiples of 5 between 10 and 40, inclusive. What is the probability that  $N + K$  is odd?
131. What is the probability of selecting a clean number from a set of integers containing all multiples of 3 between 1 and 99, inclusive?  
 (1) A clean number is an integer divisible by only 2 factors, one of which is greater than 2.  
 (2) A clean number must be odd.
132. On his drive to work, Leo listens to one of three radio stations A, B, or C. He first turns to A, if A is playing a song he likes, he listens to it; if not, he turns to B. If B is playing a song he likes, he listens to it; if not, he turns to C. If C is playing a song he likes, he listens to it; if not, he turns off the radio. For each station, the probability is 0.3 that at any given moment the station is playing a song Leo likes, on his drive to work, what is the probability that Leo will hear a song he likes?
133. A certain junior class has 1000 students and a certain senior class has 800 students. Among these students, there are 60 siblings pairs each consisting of 1 junior and 1 senior. If 1 student is to be selected at random from each class, what is the probability that the 2 students selected will be a sibling pair?  
 $\frac{3}{40000}$                        $\frac{1}{3600}$                        $\frac{9}{2000}$                        $\frac{1}{60}$                        $\frac{1}{15}$
134. Each of the 25 balls in a certain box is either red, blue, or white and has a number from 1 to 10 painted on it. If one ball is to be selected at random from the box, what is the probability that the ball selected will either be white or have an even number painted on it?  
 1). The probability that the ball will both be white and have an even number painted on it is 0.  
 2). The probability that the ball will be white minus the probability that have an eve number painted on it is 0.2
135. A certain jar contains only B black marbles, W white marbles, and R red marbles, if one marble is to be chosen at random from the jar, is the probability that the marble chosen will be red greater than the probability that marble chosen will be white?  
 1).  $r/(B+W) > w/(B+R)$                       2).  $B-W > R$
136. There are eight magazines, including 4 fashion books and 4 sports books. If three books are to be selected at random without replacement, what is the probability that at least one fashion book will be selected?  
 $\frac{1}{2}$                        $\frac{2}{3}$                        $\frac{32}{35}$                        $\frac{11}{12}$                        $\frac{13}{14}$
137. What is the probability that a number selected from (-10, -6, -5, -4, -2.5, -1, 0, 2.5, 4, 6, 7, 10) can fulfill  $(x-5)(x+10)(2x-5)=0$ ?  
 $\frac{1}{12}$                        $\frac{1}{6}$                        $\frac{1}{4}$                        $\frac{1}{3}$                        $\frac{1}{2}$

## MISCELLANEOUS QUESTIONS

### Part 1: Word Problems

- A political candidate collected \$1,749 from a fund raising dinner. If each supporter contributed at least \$50, what is the greatest possible number of contributors at the dinner?  
33                      34                      35                      36                      37
- Joan, Kylie, Lillian, and Miriam all celebrate their birthdays today. Joan is 2 years younger than Kylie, Kylie is 3 years older than Lillian, and Miriam is one year older than Joan. Which of the following could be the combined age of all four women today?  
51                      52                      53                      54                      55
- Janet is now 25 years younger than her mother Carol. If in 6 years Janet's age will be half Carol's age, how old was Janet 5 years ago?  
10                      14                      16                      19                      25
- A certain company has budgeted \$1,440 for entertainment expenses for the year, divided into 12 equal monthly allocations. If by the end of the third month, the total amount spent on entertainment was \$300, how much was the company under budget or over budget?  
\$60 under budget                      \$30 under budget                      \$30 over budget  
\$60 over budget                      \$180 over budget
- The ACME company manufactured  $x$  brooms per month from January to April, inclusive. On the first of each month, during the following May to December, inclusive, it sold  $x/2$  brooms. At the beginning of production on January 1<sup>st</sup>, the ACME company had no brooms in its inventory. If storage costs were \$1 per month per broom, approximately how much, in terms of  $x$ , did the ACME company pay for storage from May 2<sup>nd</sup> to December 31<sup>st</sup>, inclusive?  
\$ $x$                       \$3 $x$                       \$4 $x$                       \$5 $x$                       \$14 $x$
- The number of passengers on a certain bus at any given time is given by the equation  $P = -2(S - 4)^2 + 32$ , where  $P$  is the number of passengers and  $S$  is the number of stops the bus has made since beginning its route. If the bus begins its route with no passengers, how many passengers will be on the bus two stops *after* the stop where it has its greatest number of passengers?  
32                      30                      24                      14                      0
- John was 27 years old when he married Betty. They just celebrated their fifth wedding anniversary, and Betty's age is now  $7/8$  of John's. How old is Betty?  
24                      26                      28                      30                      32
- Joe needs to paint all the airplane hangars at the airport, so he buys 360 gallons of paint to do the job. During the first week, he uses  $1/4$  of all the paint. During the second week, he uses  $1/5$  of the remaining paint. How many gallons of paint has Joe used?  
18                      144                      175                      216                      250
- A certain movie star's salary for each film she makes consists of a fixed amount, along with a percentage of the gross revenue the film generates. In her last two roles, the star made \$32 million on a film that grossed \$100 million, and \$24 million on a film that grossed \$60 million. If the star wants to make at least \$40 million on her next film, what is the minimum amount of gross revenue the film must generate?  
\$110 million      \$120 million      \$130 million      \$140 million      \$150 million
- As a bicycle salesperson, Norman earns a fixed salary of \$20 per week plus \$6 per bicycle for the first six bicycles he sells, \$12 per bicycle for the next six bicycles he sells, and \$18 per bicycle for every bicycle sold after the first 12. This week, Norman earned more than twice as much as he did last week. If he sold  $x$  bicycles last week and  $y$  bicycles this week, which of the following statements must be true?  
I.  $y > 2x$                       II.  $y > x$                       III.  $y > 3$   
I only                      II only                      I and II                      II and III                      I, II, and III
- A basketball team composed of 12 players scored 100 points in a particular contest. If none of the individual players scored fewer than 7 points, what is the greatest number of points that an individual player might have scored?  
7                      13                      16                      21                      23

12. Sally has a gold credit card with a certain spending limit, and a platinum card with twice the spending limit of the gold card. Currently, she has a balance on her gold card that is  $\frac{1}{3}$  of the spending limit on that card, and she has a balance on her platinum card that is  $\frac{1}{5}$  of the spending limit on that card. If Sally transfers the entire balance on her gold card to her platinum card, what portion of her limit on the platinum card will remain unspent?  
 $\frac{11}{30}$        $\frac{29}{60}$        $\frac{17}{30}$        $\frac{19}{30}$        $\frac{11}{15}$
13. Martina earns one-sixth of her annual income during the month of June and one-eighth in August. Pam earns one-third of her annual income in June and one-fourth in August. Martina's earnings for June and August equal Pam's earnings for the same period. What portion of their combined annual income do the two girls earn during the ten months NOT including June and August?  
 $\frac{1}{8}$        $\frac{7}{24}$        $\frac{7}{18}$        $\frac{11}{18}$        $\frac{7}{8}$
14. On January 1, 2076, Lake Loser contains  $x$  liters of water. By Dec 31 of that same year,  $\frac{2}{7}$  of the  $x$  liters have evaporated. This pattern continues such that by the end of each subsequent year the lake has lost  $\frac{2}{7}$  of the water that it contained at the beginning of that year. During which year will the water in the lake be reduced to less than  $\frac{1}{4}$  of the original  $x$  liters?  
2077      2078      2079      2080      2081
15.  $\frac{3}{4}$  of all married couples have more than one child.  $\frac{2}{5}$  of all married couples have more than 3 children. What fraction of all married couples have 2 or 3 children?  
 $\frac{1}{5}$        $\frac{1}{4}$        $\frac{7}{20}$        $\frac{3}{5}$        $\frac{3}{22}$
16. Billy has an unlimited supply of the following coins: pennies (1¢), nickels (5¢), dimes (10¢), quarters (25¢), and half-dollars (50¢). On Monday, Billy bought one candy for less than a dollar and paid for it with exactly four coins (i.e., he received no change). On Tuesday, he bought two of the same candy and again paid with exactly four coins. On Wednesday, he bought three of the candies, on Thursday four of the candies, and on Friday five of the candies; each day he was able to pay with exactly four coins. Which of the following could be the price of one candy in cents?  
8¢      13¢      40¢      53¢      66¢
17. A certain violet paint contains 30 percent blue pigment and 70 percent red pigment by weight. A certain green paint contains 50 percent blue pigment and 50 percent yellow pigment. When these paints are mixed to produce a brown paint, the brown paint contains 40 percent blue pigment. If the brown paint weighs 10 grams, then the red pigment contributes how many grams of that weight?  
2.8      3.5      4.2      5      7
18. The workforce of a certain company comprised exactly 10,500 employees after a four-year period during which it increased every year. During this four-year period, the ratio of the number of workers from one year to the next was always an integer. The ratio of the number of workers after the fourth year to the number of workers after the second year is 6 to 1. The ratio of the number of workers after the third year to the number of workers after the first year is 14 to 1. The ratio of the number of workers after the third year to the number of workers before the four-year period began is 70 to 1. How many employees did the company have after the first year?  
50      70      250      350      750
19. A certain farm has a group of sheep, some of which are rams (males) and the rest ewes (females). The ratio of rams to ewes on the farm is 4 to 5. The sheep are divided into three pens, each of which contains the same number of sheep. If the ratio of rams to ewes in the first pen is 4 to 11, and if the ratio of rams to ewes in the second pen is the same as that of rams to ewes in the third, which of the following is the ratio of rams to ewes in the third pen?  
 $\frac{8}{7}$        $\frac{2}{3}$        $\frac{1}{2}$        $\frac{3}{12}$        $\frac{1}{6}$
20. The price of a bushel of corn is currently \$3.20, and the price of a peck of wheat is \$5.80. The price of corn is increasing at a constant rate of  $5x$  cents per day while the price of wheat is decreasing at a constant rate of  $(x\sqrt{2} - x)$  cents per day. What is the approximate price when a bushel of corn costs the same amount as a peck of wheat?  
\$4.50      \$5.10      \$5.30      \$5.50      \$5.60

21. At a certain college, students can major in science, math, history, or linguistics. If there are  $\frac{1}{3}$  as many science majors as there are history majors, and  $\frac{2}{3}$  as many math majors as there are history majors, how many of the 2000 students major in linguistics?
- There are as many linguistics majors as there are math majors.
  - There are 250 more math majors than there are science majors.
22. A green bucket and a blue bucket are each filled to capacity with several liquids, none of which combine with one another. Liquid A and liquid B each compose exactly 10% of the total liquid contained in the green bucket. Liquid C composes exactly 10% of the total liquid contained in the blue bucket. The entire contents of the green and blue buckets are poured into an empty red bucket, completely filling it with liquid (and with no liquid overflowing). What percent of the liquid now in the red bucket is not liquids A, B, or C?
- The total amount of liquids A, B, and C now in the red bucket is equal to 1.25 times the total amount of liquids A and B initially contained in the green bucket.
  - The green and blue buckets did not contain any of the same liquids.
23. At a certain bookstore, each notepad costs  $x$  dollars and each markers costs  $y$  dollars. If \$10 is enough to buy 5 notepads and 3 markers, is \$10 enough to buy 4 notepads and 4 markers instead?
- each notepad cost less than \$1
  - \$10 is enough to buy 11 notepads
24. If Jim earns  $x$  dollars per hour, it will take him 4 hours to earn exactly enough money to purchase a particular jacket. If Tom earns  $y$  dollars per hour, it will take him exactly 5 hours to earn enough money to purchase the same jacket. How much does the jacket cost?
- Tom makes 20% less per hour than Jim does.
  - $x + y = \$43.75$
25. Bill runs a hot dog stand, and at the end of the day he has collected an assortment of \$1, \$5, and \$10 bills. He discovers that the number of \$1, \$5, and \$10 bills that he has is in the ratio of 10 : 5 : 1, respectively. How many \$10 bills does he have?
- The dollar value of his \$1 bills equals the dollar value of his \$10 bills.
  - Bill has a total of \$225.
26. In a single row of yellow, green and red colored tiles, every red tile is preceded immediately by a yellow tile and every yellow tile is preceded immediately by a green tile. What color is the 24th tile in the row?
- The 18th tile in the row is not yellow.
  - The 19th tile in the row is not green.
27. A number of apples and oranges are to be distributed evenly among a number of baskets. Each basket will contain at least one of each type of fruit. If there are 20 oranges to be distributed, what is the minimum number of apples needed so that every basket contains less than twice as many apples as oranges?
- If the number of baskets were halved and all other conditions remained the same, there would be twice as many oranges in every remaining basket.
  - If the number of oranges were halved, it would no longer be possible to place an orange in every basket.
28. A store purchases 20 coats that each cost an equal amount and then sold each of the 20 coats at an equal price, what was the store's gross profit on the 20 coats?
- If the selling price per coat had been twice as much, the store's gross profit on the 20 coats would have been 2400
  - If the store selling price per coat had been \$2 more, the store's gross profit on the 20 coats would have been 440
29. Six countries in a certain region sent 75 representatives to an international congress, and no two countries sent the same number of representatives. Of the six countries, if country A sent the second greatest number of representatives, did country A send at least 10 representatives?
- One of the six countries sent 41 representatives to the congress.
  - Country A sent fewer than 12 representatives to the congress.
30. In a certain conference room each row of chairs has the same number of chairs, and the number of rows is 1 less than the number of chairs in a row. How many chairs are in a row?
- There is a total of 72 chairs.
  - After 1 chair is removed from the last row, there are a total of 17 chairs in the last 2 rows.

31. Store S sold a total of 90 copies of a certain book during the seven days of last week, and it sold different numbers of copies on any two of the days. If for the seven days Store S sold the greatest number of copies on Saturday and the second greatest number of the copies on Friday, did Store S sell more than 11 copies on Friday?
- (1) Last week store S sold 8 copies of the book on Thursday.
  - (2) Last week store S sold 38 copies of the book on Saturday.
32. Each person attending a fund-raising party for a certain club was charged the same admission fee, how many people attended the party?
- (1) If the admission fee had been \$0.75 less and 100 more people had attended, the club would have received the same amount in admission fees.
  - (2) If the admission fee had been \$1.50 more and 100 fewer people had attended, the club would have received the same amount in admission fees.
33. If Bob produces 36 or fewer in a week, he is paid  $X$  dollars per item. If Bob produces more than 36 items, he is paid  $X$  dollars per item for the first 36 items, and  $\frac{3}{2}$  times that amount for each additional item. How many items did Bob produce last week?
- (1) Last week Bob was paid total of \$480 for the items that he produced that week.
  - (2) This week produced 2 items more than last week and was paid a total of \$510 for the item that he produced this week.
34. Did one of three members of a certain team sell at least 2 raffle tickets yesterday?
- (1) The three members sold a total of 6 raffle tickets yesterday.
  - (2) No two of the three members sold same number of raffle tickets yesterday.
35. One kilogram of a certain coffee blend consists of  $X$  kilogram of type I and  $Y$  kilogram of type II. The cost of the blend is  $C$  dollars per kilogram, where  $C = 6.5X + 8.5Y$ . Is  $X < 0.8$ ?
- (1)  $Y > 0.15$
  - (2)  $C \geq 7.30$
36. Marta bought several pencils. If each pencil was either a 23-cent pencil or a 21-cent pencil, how many 23-cent pencils did Marta buy?
- (1) Marta bought a total of 6 pencils.
  - (2) The total value of the pencils Marta bought was 130 cents.
37. Juan bought some paperback books that cost \$8 each and hardcover books that \$25 each. If Juan bought more than 10 paperback books, how many hardcover books did he buy?
- (1) The total cost of hardcover books that Juan bought was at least \$150.
  - (2) The total cost of all books that Juan bought was less than \$260.
38. For Manufacturer M, the cost  $C$  of producing  $X$  Units of its product per month is given by  $c = kx + t$ , where  $c$  is in dollars and  $k$  and  $t$  are constants. Last month if Manufacturer M produced 1,000 units of its product and sold all the units for  $k + 60$  dollars each, what was Manufacturer M's gross profit on the 1,000 units?
- (1) Last month, Manufacturer M's revenue from the sale of the 1,000 units was 150,000.
  - (2) Manufacturer M's cost of producing 500 Units in a month is 45,000 less than its cost of producing 1,000 units in a month.
39. A computer chip manufacturer expects the ratio of the number of defective chips to the total number of chips in all future shipments to equal the corresponding ratio for shipments S1, S2, S3 and S4 combined, as shown in the following table. What is the expected number of defective chips in a shipment of 60,000 chips?

Shipment	Number of defective chips in the shipment	Total number of chips in the shipment
S1	2	5,000
S2	5	12,000
S3	6	18,000
S4	4	16,000

14

20

22

24

25

40. A certain library assesses fines for overdue books as follows. On the first day that a book is overdue, the total fine is \$0.10. For each additional day that the book is overdue the total fine is either increased by \$0.30 or



double, whichever results in the lesser amount. What is the total fine for a book on the fourth day it is overdue?

\$0.60                      \$0.70                      \$0.80                      \$0.90                      \$1.00

41. When a certain tree was first planted, it was 4 feet tall, and the height of the tree increased by a constant amount each year for the next 6 years. At the end of the 6th year, the tree was  $\frac{1}{5}$  taller than it was at the end of 4th year. By how many feet did the height of the tree increase each year?  
 $\frac{3}{10}$                        $\frac{2}{5}$                        $\frac{1}{2}$                        $\frac{2}{3}$                        $\frac{6}{5}$
42. To celebrate a colleague's retirement, the T coworkers in an office agreed to share equally the cost of a catered lunch. If the lunch cost a total of  $x$  dollars and S of the coworkers fail to pay their share, which of the following represents the additional amount, in dollars, that each of the remaining coworkers would have to contribute so that the cost of the lunch is completely paid?
43. A certain business company produced  $x$  rakes each month from November through February and shipped  $\frac{x}{2}$  at the beginning of each month from March through October. The business paid no storage cost for the rakes from November through February, but it paid storage costs of \$0.10 per rake each month from March through October for the rakes had not been shipped. In terms of  $x$ , what was the total storage cost, in dollars, that the business will paid for the rakes for the 12 months from November through October?
44. A certain company plans to sell Product X for  $p$  dollars per unit, where  $p$  is randomly chosen from all possible positive values not greater than 100. The monthly manufacturing cost for Product X (in thousands of dollars) is  $12 - p$ , and the projected monthly revenue from Product X (in thousands of dollars) is  $p(6 - p)$ . If the projected revenue is realized, what is the probability that the company will NOT see a profit on sales of Product X in the first month of sales?  
 (A) 0    (B)  $\frac{1}{100}$     (C)  $\frac{1}{25}$     (D)  $\frac{99}{100}$     (E) 1
45. Every day a certain bank calculates its average daily deposit for that calendar month up to and including that day. If on a randomly chosen day in June the sum of all deposits up to and including that day is a prime integer greater than 100, what is the probability that the average daily deposit up to and including that day contains fewer than 5 decimal places?  
 (A)  $\frac{1}{10}$                       (B)  $\frac{2}{15}$                       (C)  $\frac{4}{15}$                       (D)  $\frac{3}{10}$                       (E)  $\frac{11}{30}$
46. Three completely unmarked containers are used for measuring water. Water may be poured from one container to another, but no water may be poured outside the containers. Using nothing but the three containers and an unlimited supply of water, is it possible to measure exactly 4 gallons of water?  
 (1) The capacity of the first container is 2 gallons more than the capacity of the second container.  
 (2) The capacity of the second container is 2 gallons more than the capacity of the third container.
47. A certain cube is composed of 1000 smaller cubes, arranged 10 by 10 by 10. The top layer of cubes is removed from a face, then from the adjacent face above it, then from the adjacent face to the right of the first. The process is repeated on the same three faces in reverse order. Finally, a last layer is taken from the first face. How many smaller cubes have been removed from the larger cube?  
 (A) 488                      (B) 552                      (C) 612                      (D) 722                      (E) 900
48. Chandra and Ken are waiting in line for concert tickets. If each person takes up 2 feet of space in the line, how long is the line?  
 1) There are three people in front of Chandra and three people behind Ken  
 2) Two people are standing between Chandra and Ken
49. Nina and Teri are playing a dice game. Each girl rolls a pair of 12-sided dice, numbered with the integers from -6 through 5, and receives a score that is equal to the *negative* of the sum of the two die. (E.g., If Nina rolls a 3 and a 1, her sum is 4, and her score is -4.) If the player who gets the highest score wins, who won the game?  
 (1) The value of the first die Nina rolls is greater than the sum of both Teri's rolls.  
 (2) The value of the second die Nina rolls is greater than the sum of both Teri's rolls.
50. In the game Cako, a player is awarded one tick for every third Alb captured, and one click for every fourth Berk captured. The total score is equal to the product of clicks and ticks. If a player has a score of 77, how many Albs did he capture?  
 (1) The difference between Albs captured and Berks captured is 7.  
 (2) The number of Albs captured is divisible by 4.

51. The vertical position of an object can be approximated at any given time by the function:  $p(t) = rt - 5t^2 + b$ , where  $p(t)$  is the vertical position in meters,  $t$  is the time in seconds, and  $r$  and  $b$  are constants. After 2 seconds, the position of an object is 41 meters, and after 5 seconds the position is 26 meters. What is the position of the object, in meters, after 4 seconds?  
 (A) 24 (B) 26 (C) 39 (D) 41 (E) 45
52. A Trussian's weight, in keils, can be calculated by taking the square root of his age in years. A Trussian teenager now weighs three keils less than he will seventeen years after he is twice as old as he is now. How old is he now?  
 (A) 14 (B) 15 (C) 16 (D) 17 (E) 18
53. There are  $x$  high-level officials (where  $x$  is a positive integer). Each high-level official supervises  $x^2$  mid-level officials, each of whom, in turn, supervises  $x^3$  low-level officials. How many high-level officials are there?  
 (1) There are fewer than 60 low-level officials.  
 (2) No official is supervised by more than one person.
54. Jim went to the bakery to buy donuts for his office mates. He chose a quantity of similar donuts, for which he was charged a total of \$15. As the donuts were being boxed, Jim noticed that a few of them were slightly ragged-looking so he complained to the clerk. The clerk immediately apologized and then gave Jim 3 extra donuts for free to make up for the damaged goods. As Jim left the shop, he realized that due to the addition of the 3 free donuts, the effective price of the donuts was reduced by \$2 per dozen. How many donuts did Jim receive in the end?  
 (A) 18 (B) 21 (C) 24 (D) 28 (E) 33
55. Bobby and his younger brother Johnny have the same birthday. Johnny's age now is the same as Bobby's age was when Johnny was half as old as Bobby is now. What is Bobby's age now?  
 (1) Bobby is currently four times as old as he was when Johnny was born.  
 (2) Bobby was six years old when Johnny was born.
56. A certain clothing manufacturer makes only two types of men's blazer: cashmere and mohair. Each cashmere blazer requires 4 hours of cutting and 6 hours of sewing. Each mohair blazer requires 4 hours of cutting and 2 hours of sewing. The profit on each cashmere blazer is \$40 and the profit on each mohair blazer is \$35. How many of each type of blazer should the manufacturer produce each week in order to maximize its potential weekly profit on blazers?  
 1) The company can afford a maximum of 200 hours of cutting per week and 200 hours of sewing per week.  
 2) The wholesale price of cashmere cloth is twice that of mohair cloth.
57. Roberto has three children: two girls and a boy. All were born on the same date in different years. The sum of the ages of the two girls today is smaller than the age of the boy today, but a year from now the sum of the ages of the girls will equal the age of the boy. Three years from today, the difference between the age of the boy and the combined ages of the girls will be  
 (A) 1 (B) 2 (C) 3 (D) -2 (E) -1
58.  $x$  years ago, Cory was one fifth as old as Tania. In  $x$  years, Tania will be twice as old as Cory. What is the ratio of Cory's current age to Tania's current age?  
 (A) 7:23 (B) 9:17 (C) 5:13 (D) 3:7 (E) 11:15
59. Ten years ago, scientists predicted that the animal  $z$  would become extinct in  $t$  years. What is  $t$ ?  
 (1) Animal  $z$  became extinct 4 years ago.  
 (2) If the scientists had extended their extinction prediction for animal  $z$  by 3 years, their prediction would have been incorrect by 2 years.
60. The longevity of a certain metal construction is determined by the following formula:  $l = (7.5 - x)^4 + 8.97^c$ , where  $l$  is the longevity of the construction, in years,  $x$  is the density of the underlying material, in g/cm<sup>3</sup>, and  $c$  is a positive constant equal to 1.05 for this type of metal constructions. For what value of density,  $x$ , expressed in g/cm<sup>3</sup>, will the metal construction have minimal longevity?  
 -7.5 0 7.5 15 75

# Calculations, Exponents, Basic Algebra

- $\sqrt{\sqrt{96} + \frac{2}{5+2\sqrt{6}}} = ?$  lies between:  
 1 & 2                  2 & 3                  3 & 4                  4 & 5                  5 & 6
- List the following in increasing order from left to right:  $\sqrt[3]{2}, \sqrt[3]{5}, \sqrt[10]{10}, \sqrt[15]{30}$ ?
- $\sqrt{24+5\sqrt{23}} + \sqrt{24-5\sqrt{23}}$  lies between:  
 4 & 5                  5 & 6                  6 & 7                  7 & 8                  8 & 9
- $8^a(1/4)^b = ?$                   (1)  $b = 1.5a$                   (2)  $a = 2$
- $A, B, C, D, E, F, G,$  and  $H$  are all integers, listed in order of increasing size. When these numbers are arranged on a number line, the distance between any two consecutive numbers is constant. If  $G$  and  $H$  are equal to  $5^{12}$  and  $5^{13}$ , respectively, what is the value of  $A$ ?  
 $-24(5^{12})$                    $-23(5^{12})$                    $-24(5^6)$                    $23(5^{12})$                    $24(5^{12})$
- $(3^{5x} + 3^{5x} + 3^{5x})(4^{5x} + 4^{5x} + 4^{5x} + 4^{5x}) =$   
 $12^{5x+1}$                    $3^{15x} + 4^{20x}$                    $25^{5x}$                    $7^{35x}$                    $25^{5x+1}$
- The three-digit positive integer  $x$  has the hundreds, tens, and units digits of  $a, b,$  and  $c,$  respectively. The three-digit positive integer  $y$  has the hundreds, tens, and units digits of  $k, l,$  and  $m,$  respectively. If  $(2^a)(3^b)(5^c) = 12(2^k)(3^l)(5^m)$ , what is the value of  $x - y$ ?  
 21                  200                  210                  300                  310
- Is  $x > 10^{10}$ ?                  (1)  $x > 2^{34}$                   (2)  $x = 2^{35}$
- $\sqrt{3\sqrt{80} + \frac{3}{9+4\sqrt{5}}} = ?$   
 $2\sqrt{3\sqrt{5}}$                   3                   $3\sqrt{3}$                    $9 + 4\sqrt{5}$                    $3 + 2\sqrt{5}$
- What is the value of  $2^a 4^b$ ?  
 (1)  $a = -2b$                   (2)  $b = 4$
- If  $27^{4x+2} \times 162^{-2x} \times 36^x \times 9^{6-2x} = 1$ , then what is the value of  $x$ ?  
 -9                  -6                  3                  6                  9
- If  $(2^{2x+1})(3^{2y-1}) = 8^x 27^y$ , then  $x + y =$   
 -3                  -1                  0                  1                  3
- If  $(6^2)(44)(5^x)(20) / (8^2)(9) = 1375$ , what is the value of  $x$ ?  
 -1                  0                  1                  2                  3
- If  $5^x = y$ , what is  $x$ ?                  (1)  $y^2 = 625$                   (2)  $y^3 = 15,625$
- Wendy, Jim, and Pedro are golfing. Collectively, they have 24 golf balls. How many golf balls does Jim have?  
 (1) Jim has  $1/3$  of the number of golf balls that Wendy has.  
 (2) Pedro has  $1/2$  of the total number of golf balls.
- If  $x, y,$  and  $z$  are integers greater than 1, and  $(3^{27})(35^{10})(z) = (5^8)(7^{10})(9^{14})(x^y)$ , then what is the value of  $x$ ?  
 (1)  $z$  is prime                  (2)  $x$  is prime
- If  $4^{4x} = 1600$ , what is the value of  $(4^{x-1})^2$ ?  
 40                  20                  10                   $5/2$                    $5/4$
- If  $x$  and  $y$  are integers and  $(15^x + 15^{x+1}) / 4^y = 15^y$ , what is the value of  $x$ ?  
 2                  3                  4                  5                  Cannot be determined
- If  $3^m 3^m 3^m = 9^n$ , then  $m/n =$

- 1/3                  2/3                  1                  3/2                  3
20. If  $a = 3^{b-1}$ , what is the value of  $a + b$ ?                  (1)  $3^{b+2} = 243$                   (2)  $a = 3^{2b-4}$
21. What is the value of  $\left(\sqrt{7+\sqrt{29}} - \sqrt{7-\sqrt{29}}\right)^2$ ?  
 -26                   $2\sqrt{29}$                    $14 - 4\sqrt{5}$                   14                   $14 + 4\sqrt{5}$
22. If  $x^2/9 - 4/y^2 = 12$ , what is the value of  $x$ ?  
 (1)  $x/3 + 2/y = 6$                   (2)  $x/3 - 2/y = 2$
23. What is the value of  $(a + b)^2$ ? (1)  $a = 15/b$                   (2)  $(a - b)^2 = 4$
24. If  $x$  and  $y$  are positive and  $x^2y^2 = 18 - 3xy$ , then  $x^2 =$   
 $18 - 3y/y^3$                    $18/y^2$                    $18/y^2 + 3y$                    $9/y^2$                    $36/y^2$
25. If  $y = \sqrt{3y + 4}$ , then the product of all possible solutions for  $y$  is  
 -4                  -2                  0                  3                  6
26. If the sum of the cubes of  $a$  and  $b$  is 8 and  $a^5 - b^5 = 14$ , what is the value of  $a^3 - b^3$ ?  
 $1/4$                    $1/2$                    $5/4$                    $7/4$                   2
27. If  $x$  does not equal  $y$ , and  $xy$  does not equal 0, then when  $x$  is replaced by  $1/x$  and  $y$  is replaced by  $1/y$  everywhere in the expression  $(x + y) / (x - y)$ , the resulting expression is equivalent to  
 $-(x + y) / (x - y)$                    $(x - y) / (x + y)$                    $(x + y) / (x - y)$                    $(x + y) / (x - y)$                    $(x - y) / (x + y)$
28. If  $x$  and  $y$  are non-zero integers, and  $9x^4 - 4y^4 = 3x^2 + 2y^2$ , which of the following could be the value of  $x^2$  in terms of  $y$ ?  
 $-4y^2/3$                    $-2y^2$                    $(2y^2+1)/3$                    $2y^2$                    $6y^2/3$
29. What is the ratio of  $r$  to  $s$ ?                  (1)  $r + s = 7$                   (2)  $r^2 - s^2 = 7$
30. If there are  $x$  men and  $y$  women in a choir, and there are  $z$  more men than there are women in that choir, what is  $z$ ?  
 (1)  $x^2 - 2xy + y^2 - 9 = 0$                   (2)  $x^2 + 2xy + y^2 - 225 = 0$
31. The value of  $x$  is one quarter of  $z$ . The sum of  $x$ ,  $y$ , and  $z$  is equal to 26. If the value of  $y$  is twice the value of  $z$ , what is the largest factor of the sum of  $y$  and  $z$ ?  
 2                  3                  8                  12                  24
32. If  $2 + 5a - b/2 = 3c$ , what is the value of  $b$ ?  
 (1)  $a + c = 13$                   (2)  $-12c = -20a + 4$
33. If  $xy$  does not equal zero, what is the value of  $xy$ ?  
 (1)  $2/x + 2/y = 3$                   (2)  $x^3 - (2/y)^3 = 0$
34. The expression  $3 / (2 + \sqrt{3})$  is equal to:  
 $6 + 3\sqrt{3}$                    $6 - 3\sqrt{3}$                    $(6 + 3\sqrt{3}) / 7$                    $(6 - 3\sqrt{3}) / 7$                    $1.5 + \sqrt{3}$
35. If  $(1/5)^m \times (1/4)^{18} = 1/2 \times (10)^{35}$ , then  $m =$ ?
36. If  $5^{21} \times 4^{11} = 2 \times 10^n$ , what is the value of  $n$ ?  
 11                  21                  22                  23                  32
37. Which of the following best approximates the value of  $q$  if  $5^{28} + 3^{11} = 5^q$ ?  
 39                  30                  28                  27                  17
38. What is the value of  $(2^x + 2^x) / 2^y$ ?                  (1)  $x - y = 8$                   (2)  $x/y = -3$
39. If  $x$  is not equal to  $y$  and if  $\sqrt{x} = y$ , what is the value of  $y^3$ ?  
 (1)  $x = yx$                   (2)  $x^3 = 8$

40. Is  $Y$  greater than  $7/11$ ? (1)  $1/5 < Y < 11/12$  (2)  $2/9 < Y < 8/13$
41. If  $(\sqrt{x} + \sqrt{y}) / (x - y) = (2\sqrt{x} + 2\sqrt{y}) / [x + 2\sqrt{(xy)} + y]$ , what is the ratio of  $x$  to  $y$ ?  
 $\frac{1}{2} \quad 2 \quad 4 \quad 7 \quad 9$
42. Is  $pq = 1$ ? (1)  $pqp = p$  (2)  $qpq = q$
43.  $(16x^4 - 81y^4) / (2x + 3y) = 12x^2 + 27y^2$  and  $4x + 3y = 9$ , what is  $x$ ?  
 $1 \quad 1.5 \quad 2 \quad 3 \quad 4$
44. If  $f(x) = ax^4 - 4x^2 + ax - 3$ , then  $f(b) - f(-b)$  will equal:  
 $0 \quad 2ab \quad 2ab^4 - 8b^2 - 6 \quad -2ab^4 + 8b^2 + 6 \quad 2ab^4 - 8b^2 + 2ab - 6$
45. If  $p \& q = p^2 + q^2 - 2pq$ , for what value of  $q$  is  $p \& q$  equal to  $p^2$  for all values of  $p$ ?  
 $-2 \quad -1 \quad 0 \quad 1 \quad 2$
46. If  $t$  and  $u$  are positive integers, what is the value of  $t^{-2u-3}$ ?  
 (1)  $t^{-3u-2} = 1/36$  (2)  $t^{u-1} = 1/6$
47. If  $a$  and  $b$  are different values and  $a - b = \sqrt{a} - \sqrt{b}$ , then in terms of  $b$ ,  $a$  equals:  
 $\sqrt{b} \quad b \quad b - 2\sqrt{b} + 1 \quad b + 2\sqrt{b} + 1 \quad b^2 - 2b\sqrt{b} + b$
48. What is the value of  $(a! + b!)(c! + d!)$ ?  
 (1)  $b!d! = 4(a!c!)$   
 (2)  $60(b!c!) = (b!d!)$
49. If  $f(x) = 125/x^3$ , what is the value of  $f(5x) / f(x/5)$  in terms of  $f(x)$ ?  
 (A)  $(f(x))^2$  (B)  $f(x^2)$  (C)  $(f(x))^3$  (D)  $f(x^3)$  (E)  $f(125x)$
50. If  $[3(ab)^3 + 9(ab)^2 - 54ab] / [(a-1)(a+2)] = 0$ , and  $a$  and  $b$  are both non-zero integers, which of the following could be the value of  $b$ ?  
 I. 2 II. 3 III. 4  
 (A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III
51. If the reciprocals of two consecutive integers are added to one another, what is the sum in terms of the greater integer  $x$ ?
52. What is the value of  $y + x^3 + x$ ? (1)  $y = x(x-3)(x+3)$  (2)  $y = -5x$
53. If  $3x - 2y - z = 32 + z$  and  $\sqrt{(3x)} - \sqrt{(2y + 2z)} = 4$ , what is the value of  $x + y + z$ ?  
 (A) 3 (B) 9 (C) 10 (D) 12 (E) 14
54. If  $z$  is not equal to zero, and  $z = \frac{\sqrt{6zs - 9s^2}}{s}$ , then  $z$  equals:  
 $s \quad 3s \quad 4s \quad -3s \quad -4s$
55. If  $3^k + 3^k = \{3^9\}^{3^9} - 3^k$ , then  $k = ?$
56. If  $a^{\frac{2}{3}} - b^{\frac{2}{3}} = 12$ , then  $\sqrt[3]{a} + \sqrt[3]{b} = ?$   
 (1)  $\sqrt[3]{a} = \sqrt[3]{b} + 2$  (2)  $a = 64$
57. If  $a$ ,  $b$ ,  $x$  and  $y$  are positive integers, what is the value of  $a - b$ ?  
 (1)  $x^a = x^b + x^b + x^b$  (2)  $y^a = y^b + y^b + y^b + y^b$
58. If  $(a+b)^x = a^x + y(a^{x-1}b^{x-4}) + z(a^{x-2}b^{x-3}) + z(a^{x-3}b^{x-2}) + y(a^{x-4}b^{x-1}) + b^x$ , what is the value of  $yz$ ?  
 (A) 24 (B) 30 (C) 36 (D) 42 (E) 50
59.  $x$  and  $y$  are positive integers. If  $5^x - 5^y = (2^{y-1})(5^{x-1})$ , what is the value of  $xy$ ?

(A) 48                      (B) 36                      (C) 24                      (D) 18                      (E) 12

60. For a three-digit number  $xyz$ , where  $x$ ,  $y$ , and  $z$  are the digits of the number,  $f(xyz) = 5^x 2^y 3^z$ . If  $f(abc) = 3 \cdot f(def)$ , what is the value of  $abc - def$ ?  
 (A) 1                      (B) 2                      (C) 3                      (D) 9                      (E) 27

61. If  $x$ ,  $y$ , and  $z$  are integers and  $2^x 5^y z = 0.00064$ , what is the value of  $xy$ ?  
 (1)  $z = 20$                       (2)  $x = -1$

62. If  $x$  is a non-zero integer, what is the value of  $x$ ?  
 (1)  $x = 2$                       (2)  $(128^x)(6^{x+y}) = (48^2x)(3^{-x})$

63. If  $n$  is an integer and  $f(n) = f(n-1) - n$ , what is the value of  $f(4)$ ?  
 (1)  $f(3) = 14$                       (2)  $f(6) = -1$

64. If  $\#p\# = ap^3 + bp - 1$  where  $a$  and  $b$  are constants, and  $\#-7\# = 3$ , what is the value of  $\#7\#$ ?  
 5                      0                      -2                      -3                      -5

65. Let  $f(x) = x^2 + bx + c$ . If  $f(6) = 0$  and  $f(-3) = 0$ , then  $b + c =$   
 18                      15                      -15                      -21                      -24

66. If  $\sqrt{4+x^{1/2}} = \sqrt{x+2}$ , then  $x$  could be equal to which of the following?  
 -1                      0                      1                      4                      cannot be determined.

67. If  $6xy = x^2y + 9y$ , what is the value of  $xy$ ?  
 (1)  $y - x = 3$                       (2)  $x^3 < 0$